Optimal Bounded Control and Relevant Dynamic Analysis of Certain Stochastic Systems

D.V. Iourtchenko
Assistant Professor
University of Miami

Stochastic optimal control problems can be found in a variety of engineering applications: airspace, earthquake, civil and mechanical, ocean engineering, manufacturing processes and robots, tracking and guidance systems, land vehicles subjected to environmental uncertainties, etc. The presentation discusses problems of stochastic optimal control with bounded in magnitude control force $|u| \le R$. The problems are rarely addressed due to their mathematical complexity therefore some of the fundamental engineering problems have not been solved until very recently.

The first part of the presentation is devoted to the new Hybrid Solution Method, developed recently by the author and his colleagues. The method allows constructing solutions to certain types of stochastic optimal control problems. The method consists of two steps: first, the exact analytical solution to the corresponding Hamilton-Jacobi-Bellman (HJB) equation is found in a certain "outer" domain. Then, this solution is used to solve the HJB equation numerically within the remaining "inner" domain. Results for systems with external control have demonstrated for the first time that an optimal control law for minimizing steady-state mean system response energy is a dry friction law $R\,sign(v)$. Results for systems with internal or parametric control have demonstrated for the first time that an optimal control law for minimizing steady-state mean system response energy is $R\,sign(xv)$, where x,v are the system's displacement and velocity correspondingly.

The second part of the presentation is devoted to dynamic analysis of optimally controlled stochastic systems. As it has been shown, implementation of the optimal control law makes the corresponding dynamic system strongly nonlinear (signum type nonlinearity). For these systems the new Direct Energy Balance (DEB) Method is proposed, which provides certain estimations about mean system response energy. The DEB method has also been successfully applied to vibroimpact systems with inelastic impacts. It has been shown, by comparing analytical results with results of Monte-Carlo simulation, that the proposed DEB method provides superior accuracy over the existing Stochastic Averaging Method for the whole range of the parameter R.