A Monte Carlo approach for efficient estimation of extreme response statistics

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Introduction

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- Recent years have seen the appearance of importance sampling techniques also for dynamical systems, but these are fairly involved and are not likely to become widely used in practical applications.
- Are standard Monte Carlo methods really useless in this context?
- NOT QUITE!

The Response Process

- The equations of motion for the dynamic system considered is assumed to be of the form

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M denotes a generalized $n \times n$ mass matrix,
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  $\mathbf{F}(t)$ denotes a stochastic loading process.
- Hence the solution $\mathbf{X}(t)$ is also a stochastic vector process.
- For specific prediction purposes, it is usually the extreme values of one, or possibly a combination of several, of the component processes of $\mathbf{X}(t)$ that is sought. For simplicity, denote it by $X(t)$.

The Mean Upcrossing Rate

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Under suitable regularity conditions on the response process the following formula obtains

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where $f_{X(t)\dot{X}(t)}(\cdot, \cdot)$ denotes the joint PDF of $X(t)$ and $\dot{X}(t) = dX(t)/dt$.

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$$\nu^+(\xi; t) = \int_0^\infty s\, f_{\dot{X}(t)|X(t)}(s|\xi)\, ds\, f_{X(t)}(\xi) = \mathrm{E}[\dot{X}(t)^+|X(t) = \xi]\, f_{X(t)}(\xi),$$

where $\dot{X}^+ = \max(\dot{X}, 0)$.

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Note that the parameter of the Poisson distribution is $\text{E}[N^+(\xi;0,T)] = \int_0^T \nu^+(\xi;t)\,dt$.

The Mean Upcrossing Rate

It is expedient to rewrite the extreme value distribution as

$$F_{M(T)}(\xi) = \text{Prob}(M(T) \leq \xi) = \exp\left\{-\overline{\nu}^{+}(\xi)\, T\right\},$$

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The averaged mean upcrossing rate $\overline{\nu}^{+}(\xi)$ is conveniently estimated from simulated response time histories.

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For a suitable number k, e.g. $k \geq 20 - 30$, a good approximation of the 95 % confidence interval for the value $\overline{\nu}^+(\xi)$ is

$$\text{conf. band}(\xi) = \hat{\overline{\nu}}^+(\xi) \pm 1.96\,\hat{s}(\xi)/\sqrt{k}$$

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The empirical standard deviation $\hat{s}(\xi)$ is given as

$$\hat{s}(\xi)^2 = \frac{1}{k-1} \sum_{j=1}^{k} \Big(\frac{n_j^+(\xi; 0, T)}{T} - \hat{\overline{\nu}}^+(\xi) \Big)^2$$

Mean Upcrossing Rate versus PDF - stationary case

- The PDF $f_X(x)$ of $X(t)$ is written as

$$f_X(x) = \exp\{-\alpha(x)\},$$

where $\alpha(x)$= a well-behaved function that is strictly increasing for increasing x for $x \geq x_0$ for some x_0.

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- Now we can write

  $$\nu_X^+(x) = q\,\exp\{-\alpha(x) + \delta(x)\}\,,$$

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- $q\,\exp\{-\alpha(x)\} = q\,f_X(x)$ expresses the mean upcrossing rate for the case with independent $X(t)$ and $\dot{X}(t)$.

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- Plotting $\ln \nu_X^+(x)$ versus $\ln f_X(x)$ will then clearly show to what extent $|\delta(x)|$ is dominated by $\alpha(x)$ as $x \to \infty$.

Extrapolation of Mean Upcrossing Rate

- Assumption:

$$\alpha(x) = a(x-b)^c - d(x)\,,\; x \geq x_0,$$

where a, b and c are suitable constants, and $d(x)$ is a function of much slower increase than $\alpha(x)$.

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- Hence, we assume that

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- The particular choice for the function $\alpha(x)$ reflects the basic assumption of an asymptotic Gumbel distribution of the extremes.

Extrapolation of Mean Upcrossing Rate

- It follows that

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- Choice of initial value $\tilde{q}_0$ for $\tilde{q}(x)$ would be based on looking at the ratio $\nu_X^+(x)/f_X(x) = \tilde{q}(x)\ \exp\{-d(x)\}$ for large x.
- Practical solution: $\tilde{q}_0 = \langle\nu_X^+(x)/f_X(x)\rangle$ (tail average), followed by optimization wrt b.

Numerical Examples - Duffing oscillator

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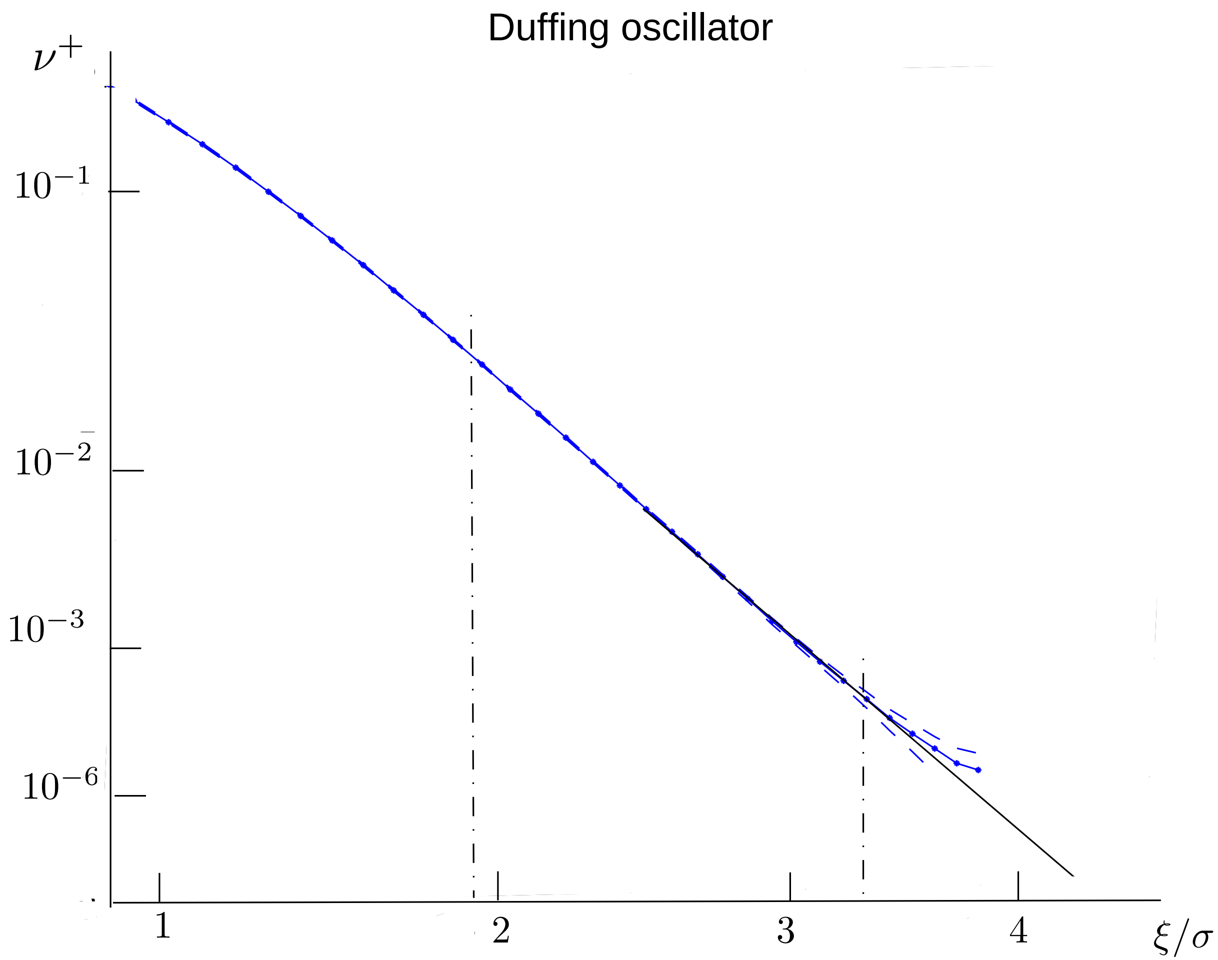
- $f_{X\dot{X}} = f_X\, f_{\dot{X}}$ is known in closed form.

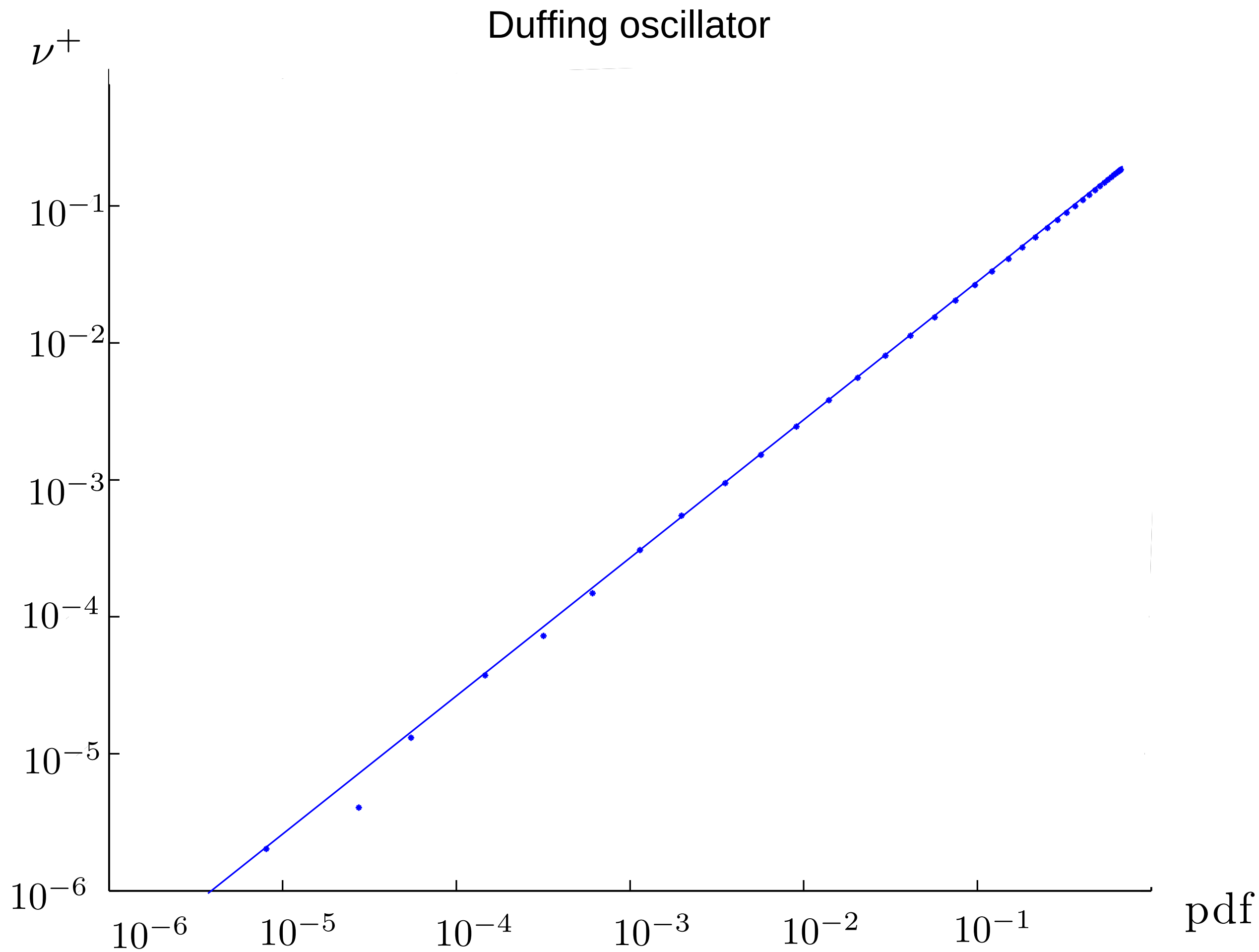
Numerical Examples - Duffing oscillator

Upcrossing rates estimated from Monte Carlo simulations ($*$) with 95% confidence band ($--$) versus analytical results (—) for the mean upcrossing rate.



Numerical Examples - Duffing oscillator

Monte Carlo ($*$) and analytical (—) results the mean upcrossing rate versus PDF on the log scale. Slope = 1.0



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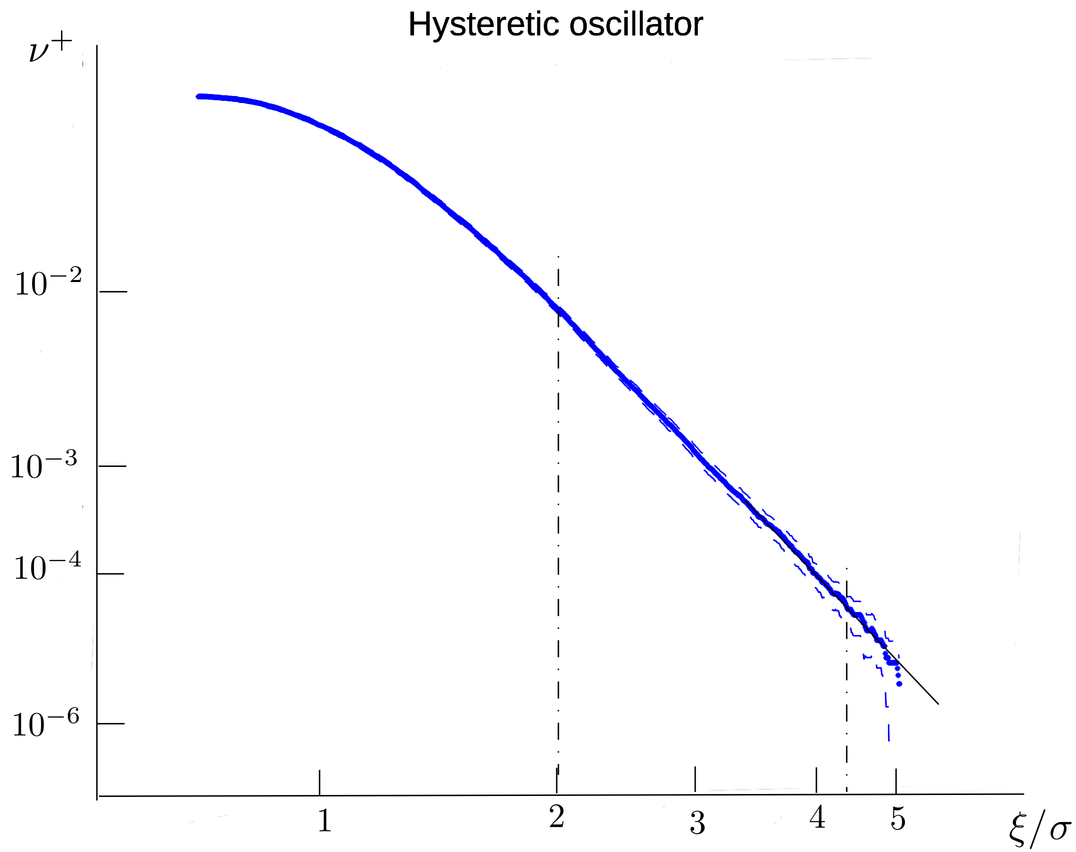
- $f_{X\dot{X}}$ unknown.

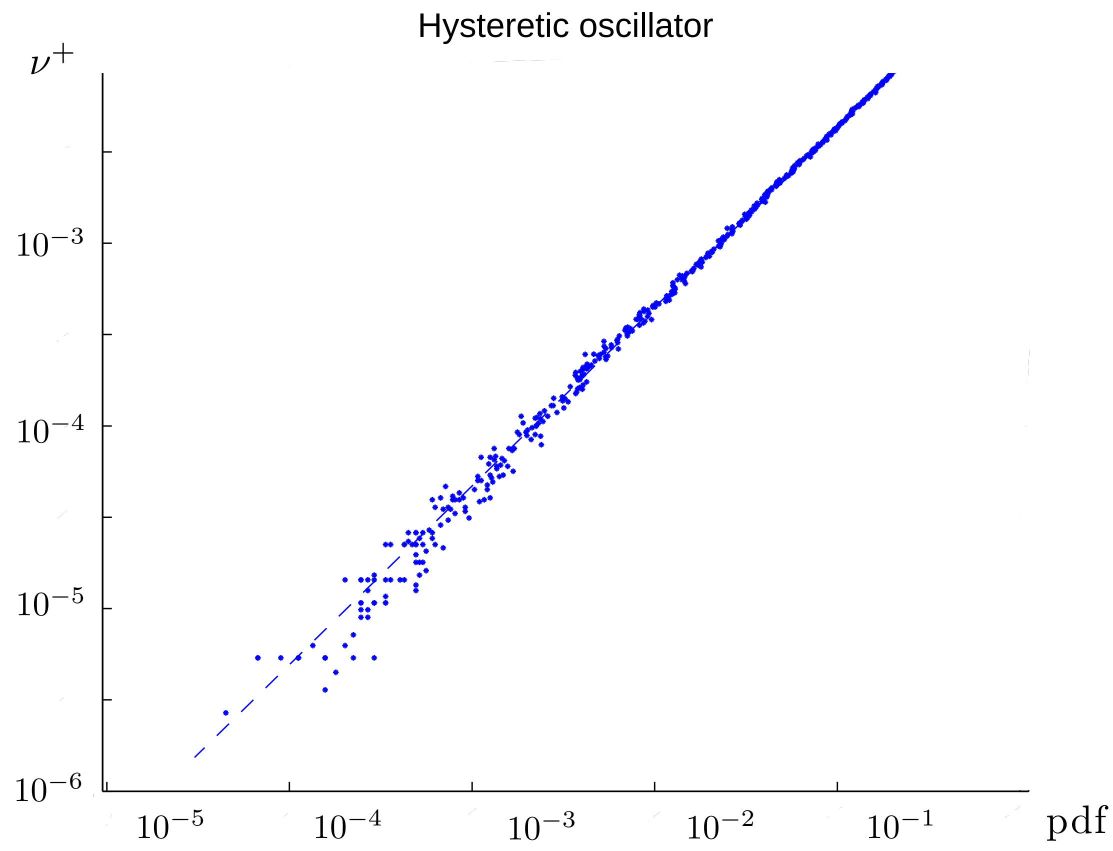
Numerical Examples - Hysteretic oscillator

Monte Carlo results for the mean upcrossing rate, 100 realizations (*) along with 95% confidence bands (– –) versus 50000 realizations (—).



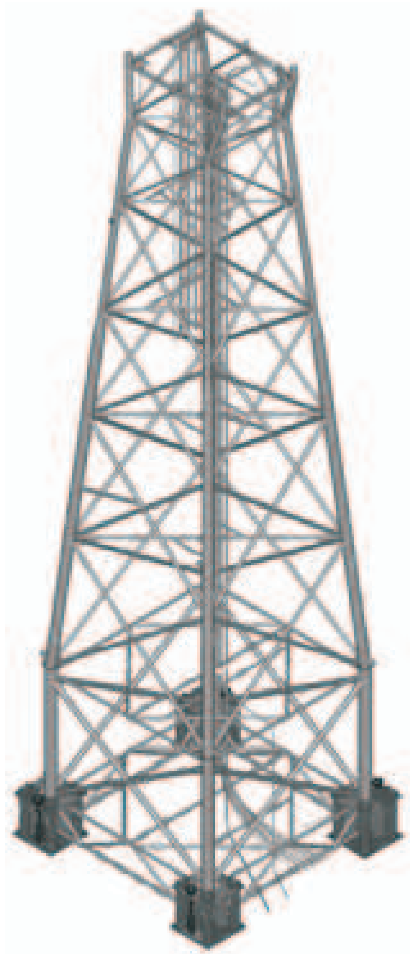
Numerical Examples - Hysteretic oscillator

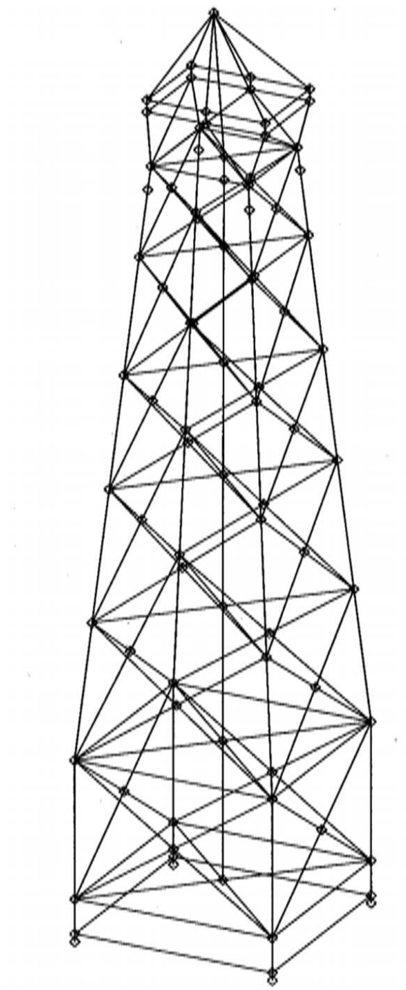
Monte Carlo results for the mean upcrossing rate versus PDF, 100 realizations (*) versus 50000 realizations (−−). Slope = 1.02



Numerical Examples - Jacket structure

The Kvitebjørn jacket platform with the superstructure removed.





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- $\mathbf{Q} = (Q(t, \mathbf{x}_1), \ldots, Q(t, \mathbf{x}_N))^T$, where $Q(t, \mathbf{x}_k) = F_{in}(t, \mathbf{x}_k) + F_d(t, \mathbf{x}_k)$, $k = 1, \ldots, N$ and $-d = z_1 \leq z_k \leq z_N = L - d$, where $d = 190$ m is the water depth and $L = 216$ m is the jacket support height.

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- The drag force components

$$F_d(t, \mathbf{x}_k) = k_d \left(U(t, \mathbf{x}_k) + U_c \right) | U(t, \mathbf{x}_k) + U_c |$$

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$$F_{in}(t,\mathbf{x}_k) = k_m \dot{U}(t,\mathbf{x}_k)$$

- The drag force components

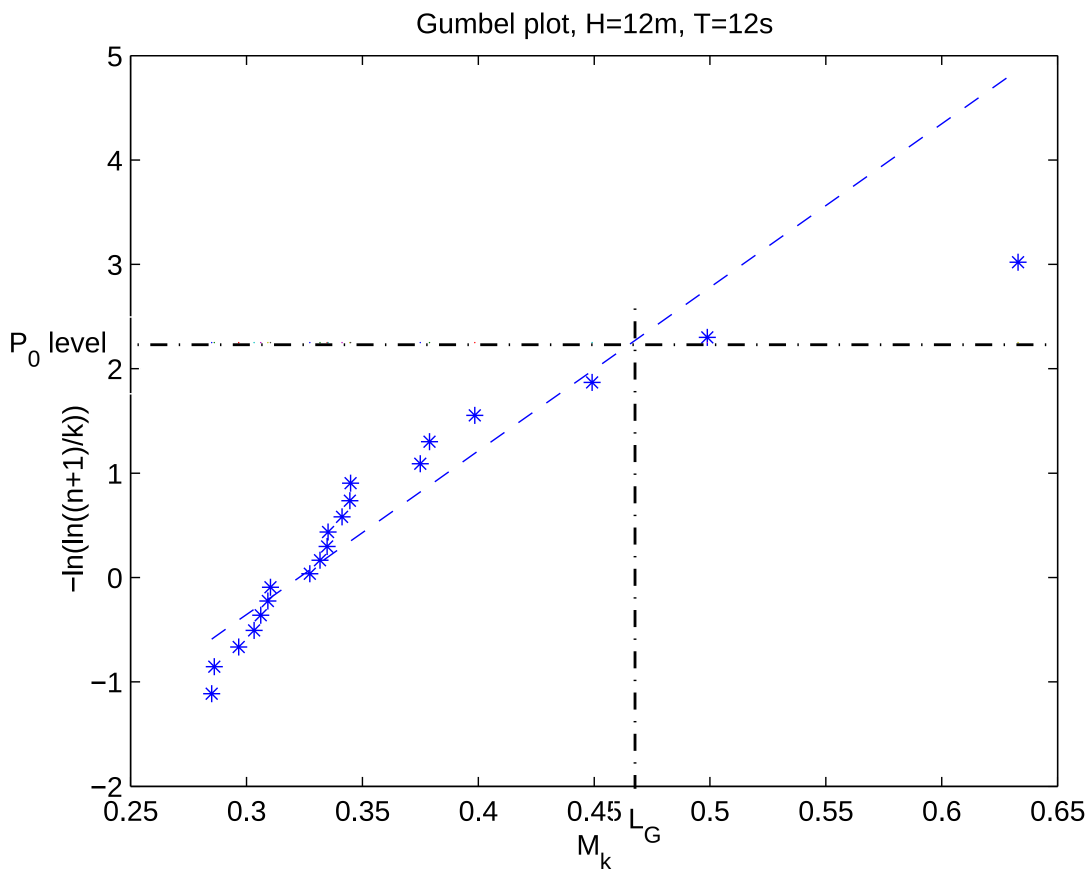
$$F_d(t,\mathbf{x}_k) = k_d \left(U(t,\mathbf{x}_k) + U_c\right)|U(t,\mathbf{x}_k) + U_c|$$

-

$$k_m = C_m \rho \pi D^2/4, \; k_d = C_d \rho D/2$$

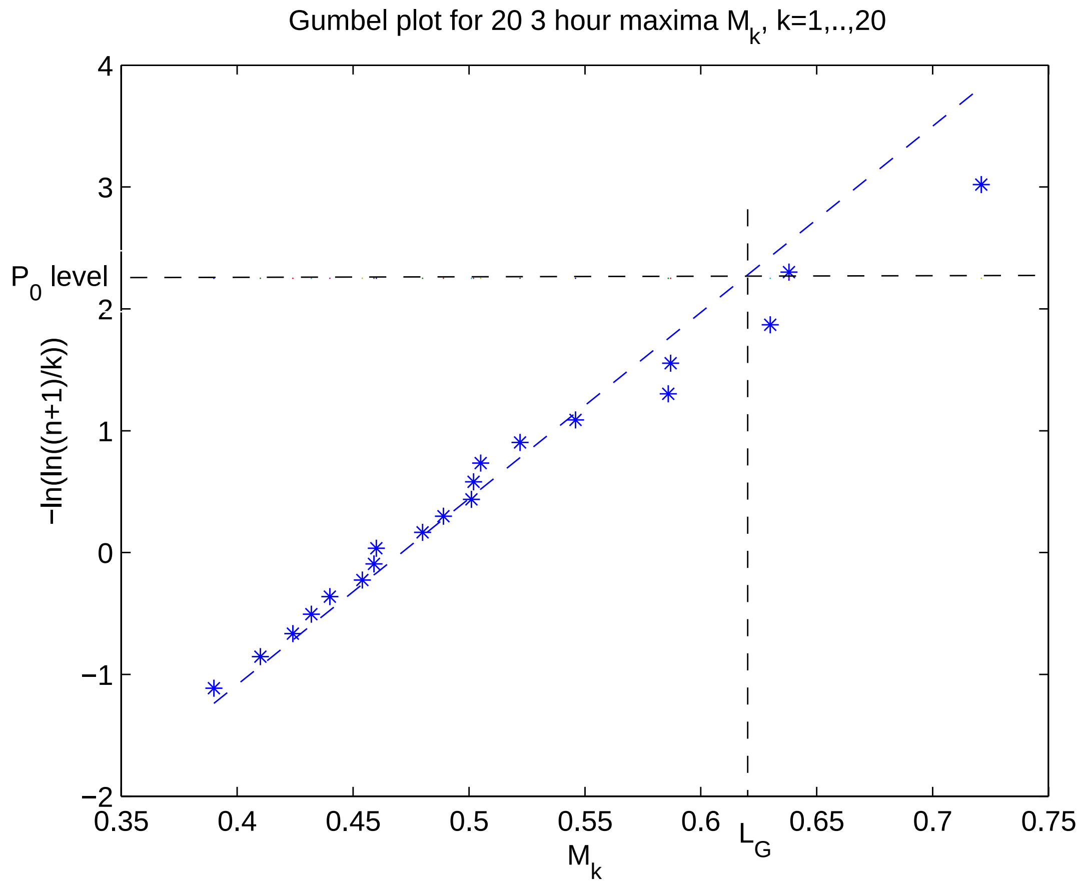
Numerical Examples - Jacket structure

Gumbel plot of 20 simulated 3 hour extremes with fitted Gumbel distribution. Sea state with $H_s = 12$ m, $T_p = 12$ s.



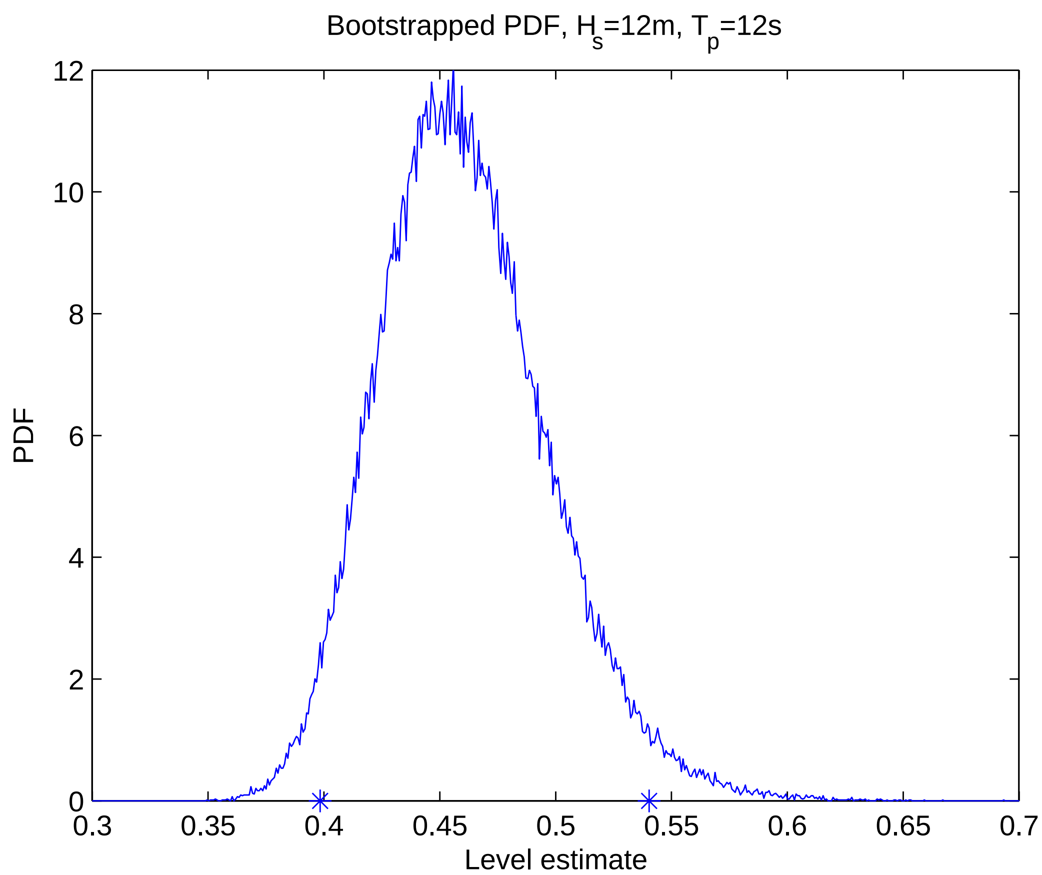
Numerical Examples - Jacket structure

Gumbel plot of 20 simulated 3 hour extremes with fitted Gumbel distribution. Sea state with $H_s = 14.7$ m, $T_p = 15$ s.



Numerical Examples - Jacket structure

Empirical PDF of the 90% fractile value based on samples of size 20 for the sea state with $H_s = 12$ m, $T_p = 12$ s.



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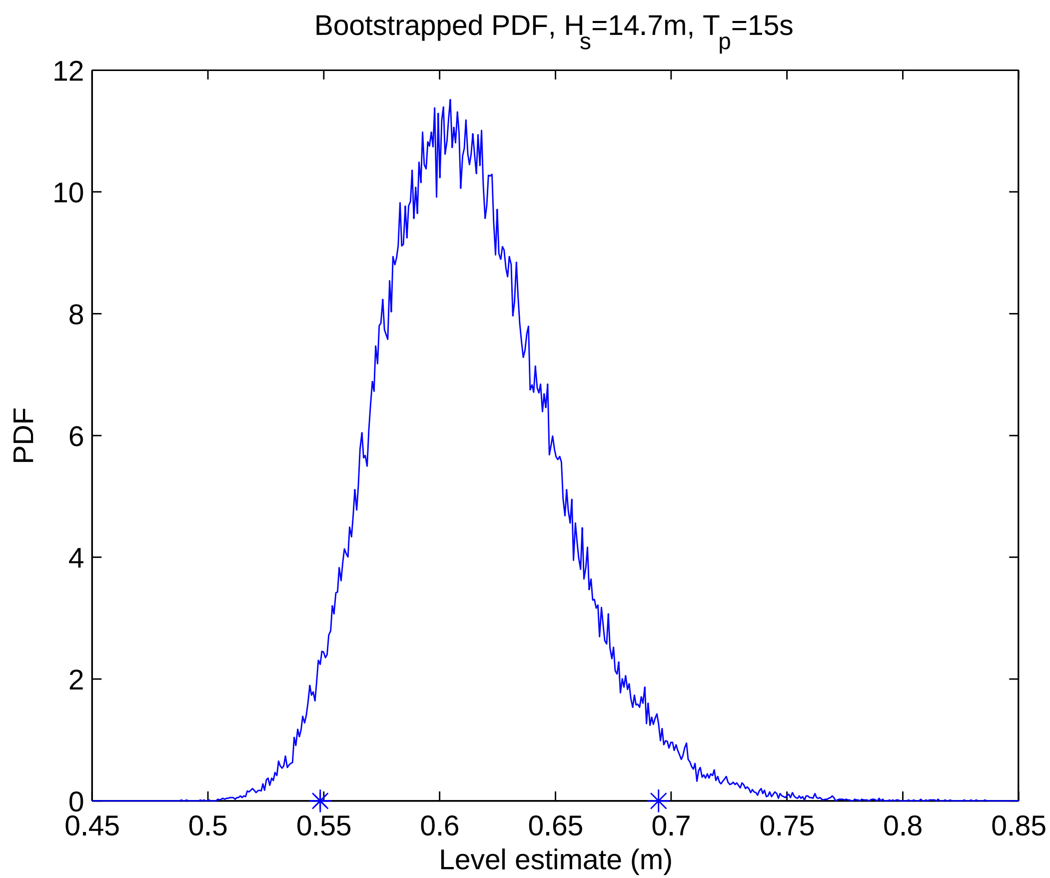
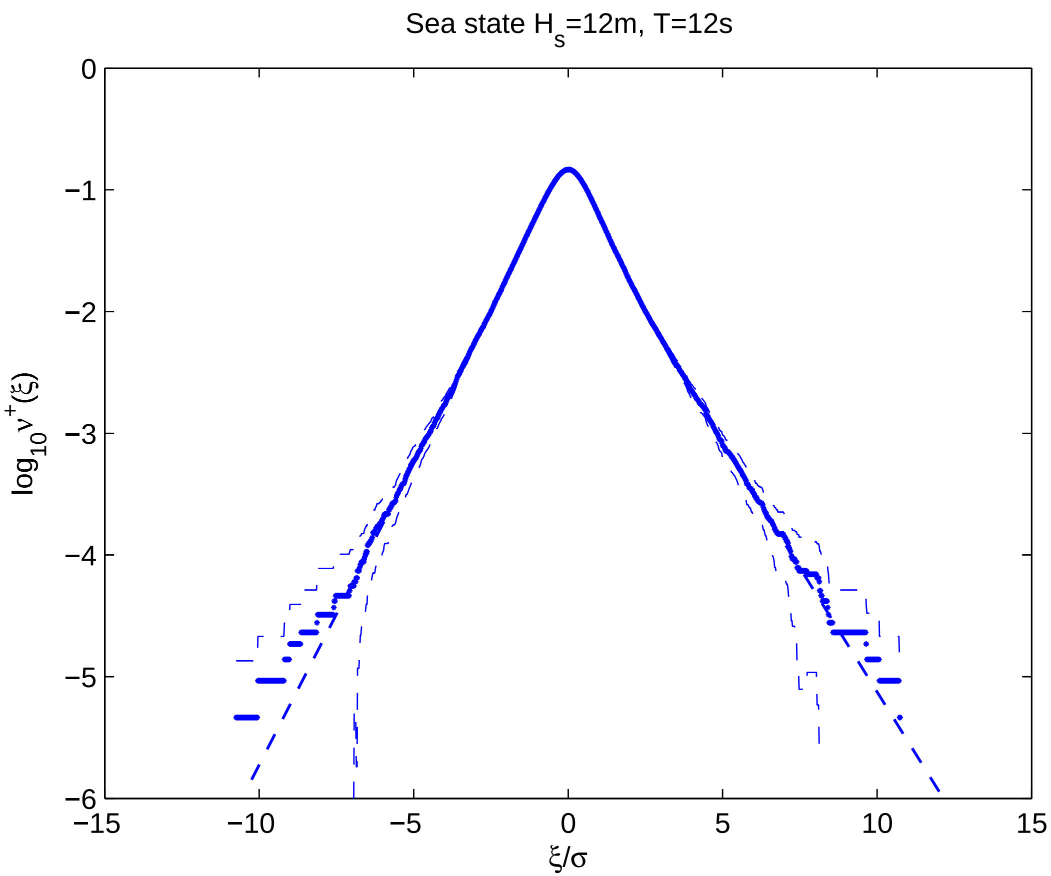


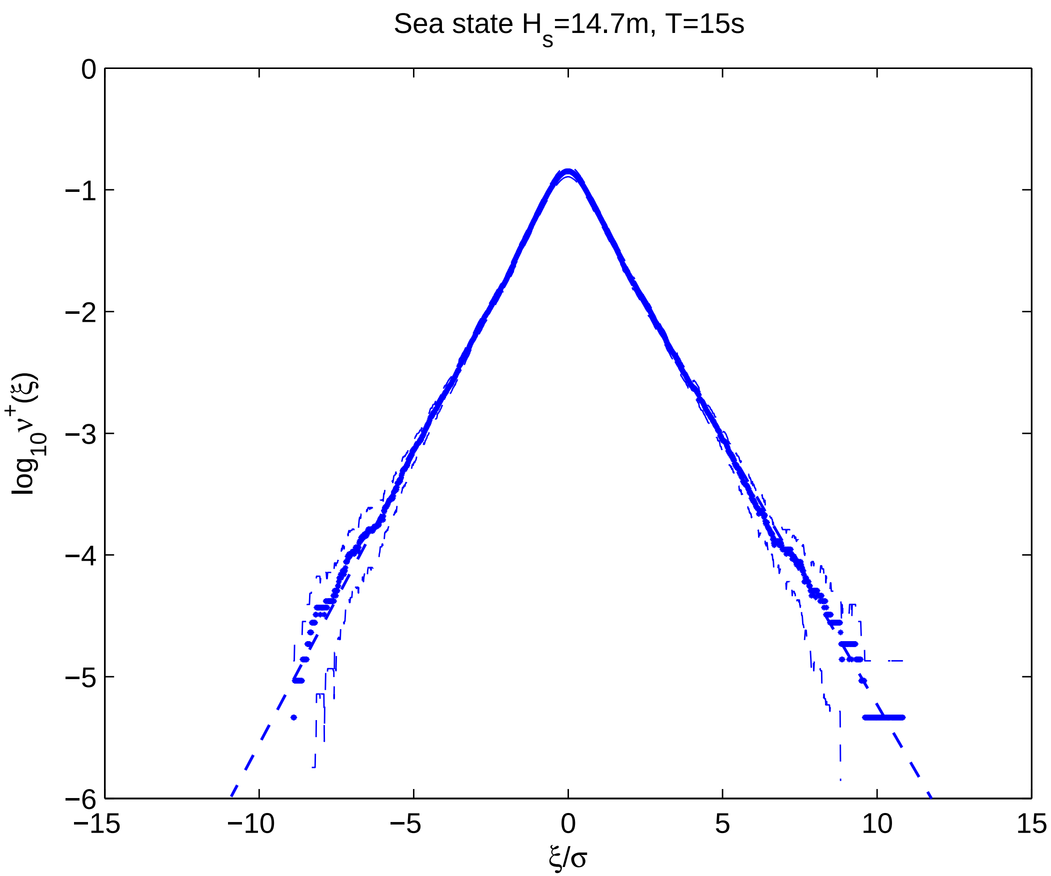
Numerical Examples - Jacket structure

Mean upcrossing rate statistics along with 95% confidence bands (−−) for the sea state with $H_s = 12$ m, $T_p = 12$ s, $\sigma = 0.047$ m. $*$: Monte Carlo; $- \; - \; -$: linear fit.



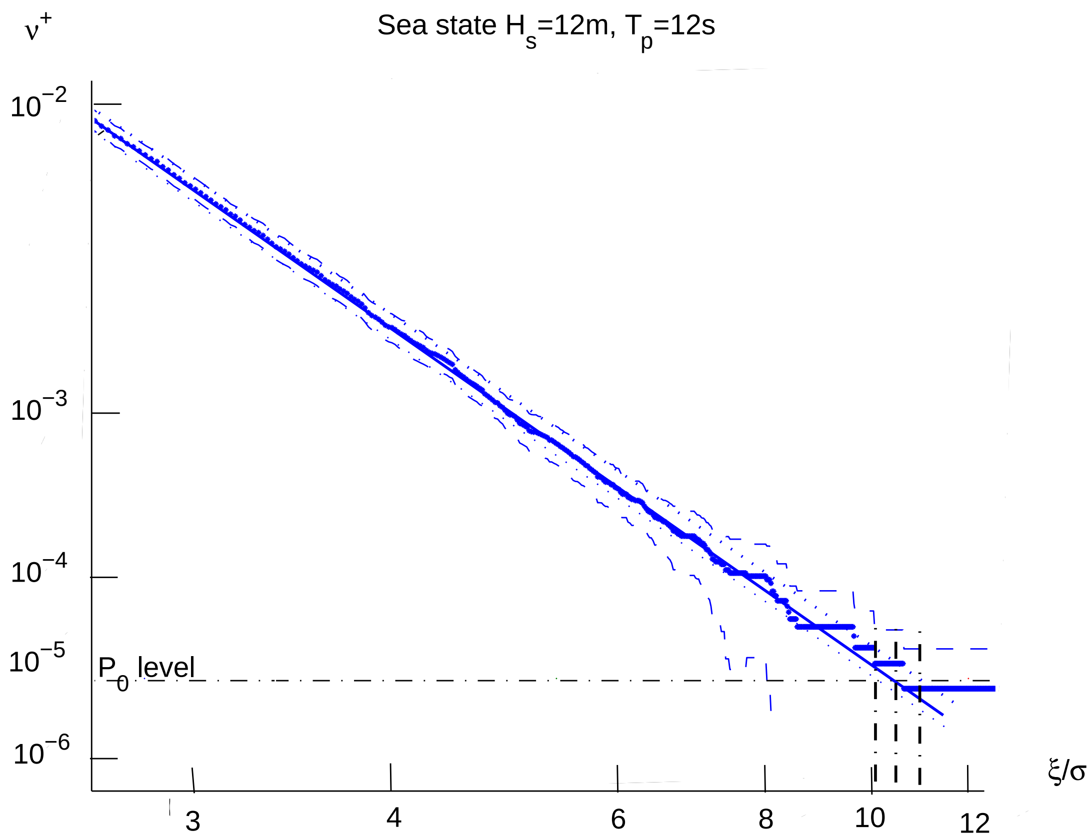
Numerical Examples - Jacket structure

Mean upcrossing rate statistics along with 95% confidence bands (−−) for the sea state with $H_s = 14.7$ m, $T_p = 15$ s, $\sigma = 0.066$ m. $*$: Monte Carlo; $- \; - \; -$: linear fit.



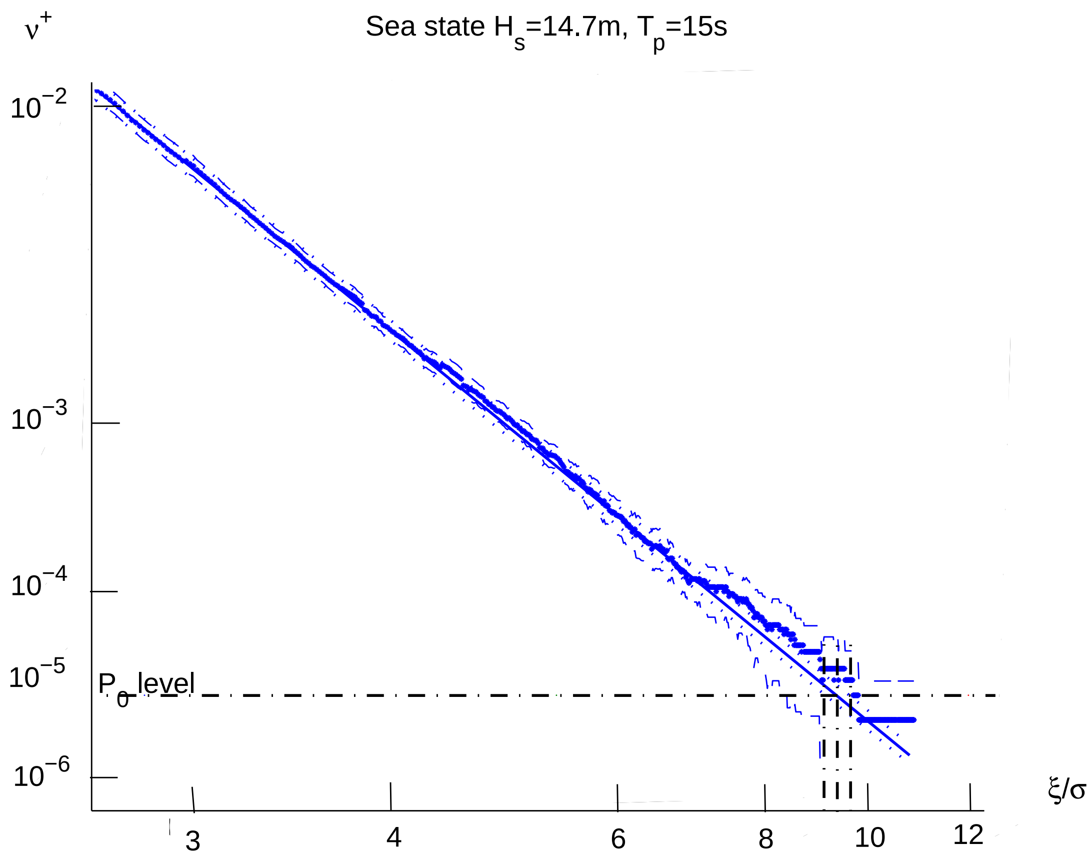
Numerical Examples - Jacket structure

Transformed plot along with 95% confidence bands (−−) for the sea state with $H_s = 12$ m, $T_p = 12$ s, $\sigma = 0.047$ m. $*$: Monte Carlo; —— : linear fit, $q = 0.04$, $b = 1.4\sigma$.



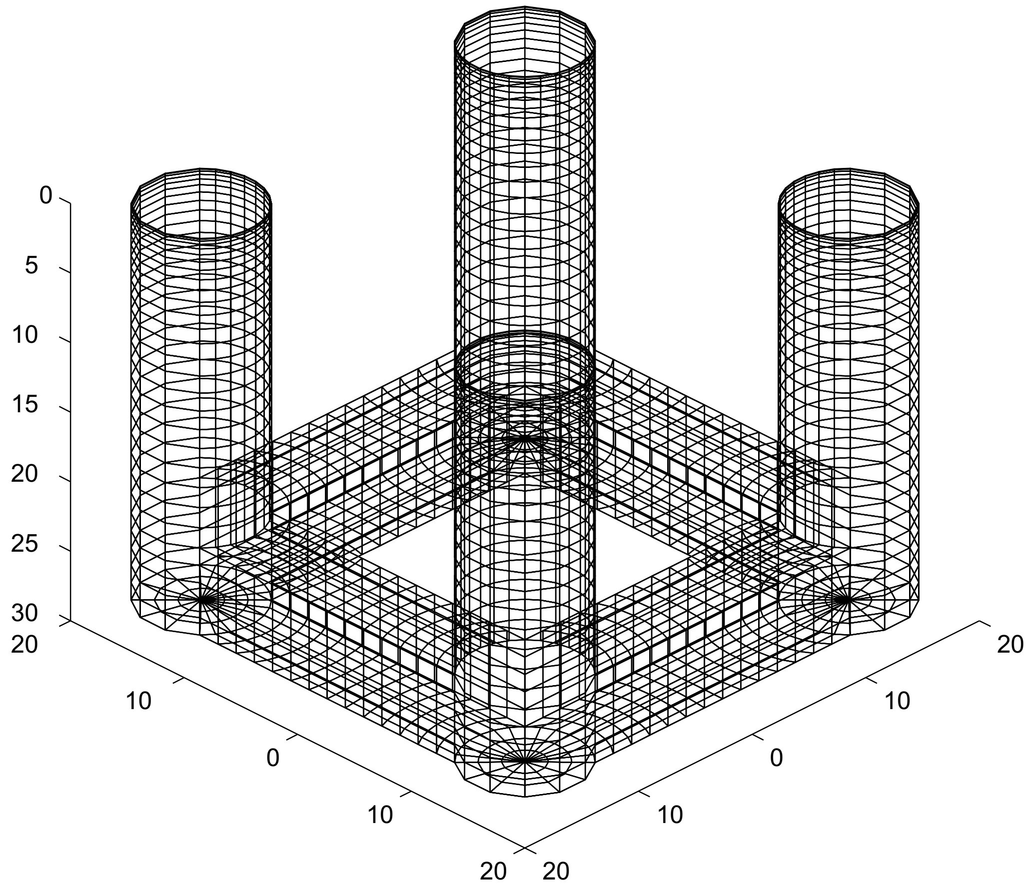
Numerical Examples - Jacket structure

Transformed plot along with 95% confidence bands (– –) for the sea state with $H_s = 14.7$ m, $T_p = 15$ s, $\sigma = 0.066$ m. $*$: Monte Carlo; —— : linear fit, $q = 0.06$, $b = 0.9\sigma$.



Numerical Examples - TLP

Sketch of submerged part of TLP.



Numerical Examples - TLP

- The equation of motion for the horizontal excursions of the TLP is

$$\mathbf{M}\ddot{\mathbf{Z}}(t) + \mathbf{D}(t)\,\dot{\mathbf{Z}}(t) + \mathbf{C}(\dot{\mathbf{Z}}(t)) + \mathbf{K}(\mathbf{Z}(t)) = \mathbf{F}(t)$$

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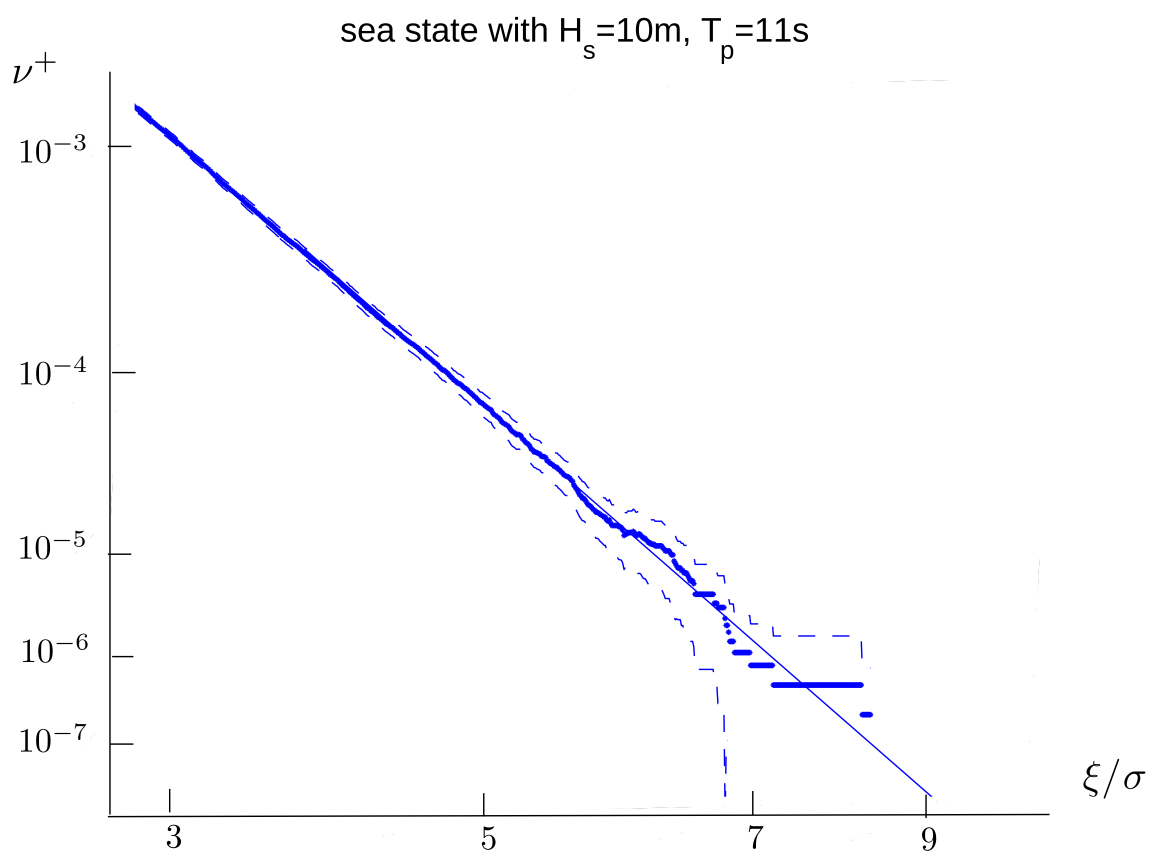
$$\ddot{Z} + 2\omega_e(\zeta_0 + \tilde{c}F_2(t))\dot{Z} + \omega_e^2(Z + \tilde{\varepsilon}Z^3) = \frac{1}{M}\big(F_1(t) + F_2(t)\big)$$

Numerical Examples - TLP

Crossing rates by Monte Carlo simulation (*) with 95% confidence bands (– –) and by saddle point integration (——) for the case of linear dynamics ($\tilde{c} = \tilde{\varepsilon} = 0$). Sea state with $H_s = 10$ m, $T_p = 11$ s.



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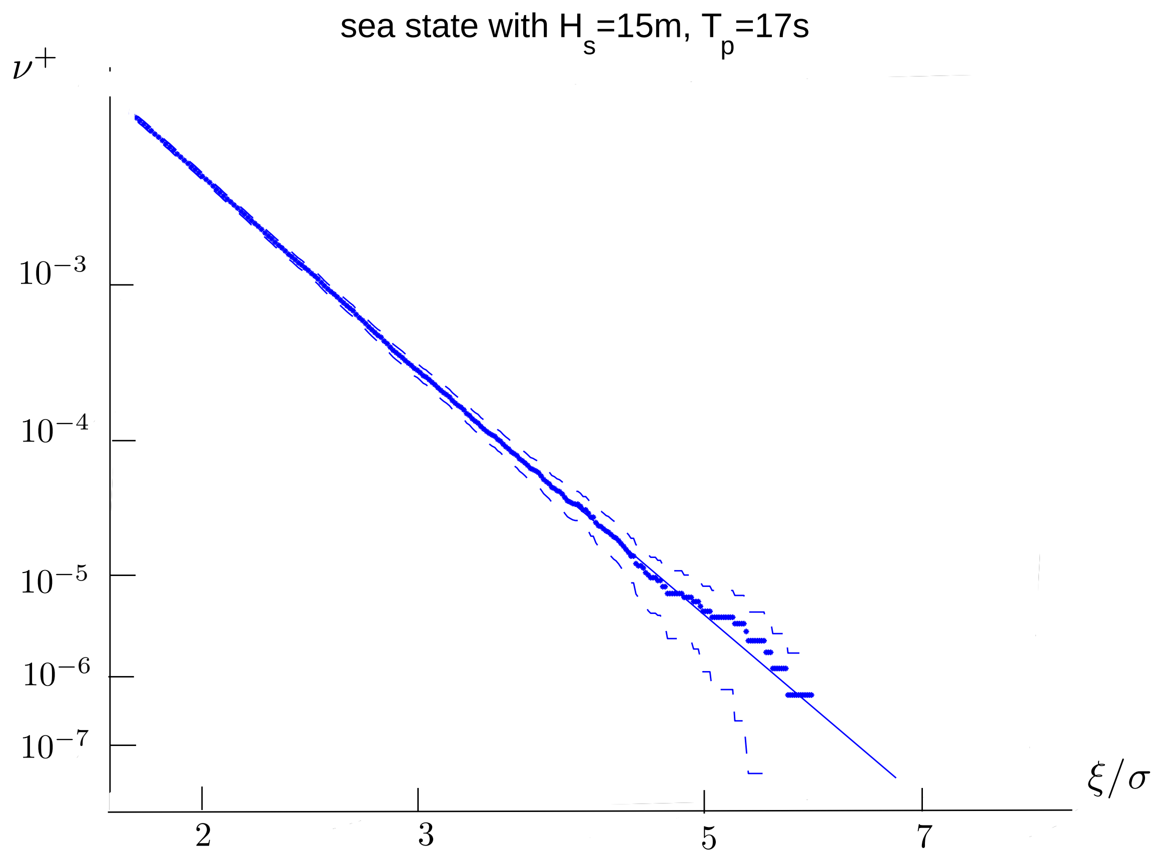
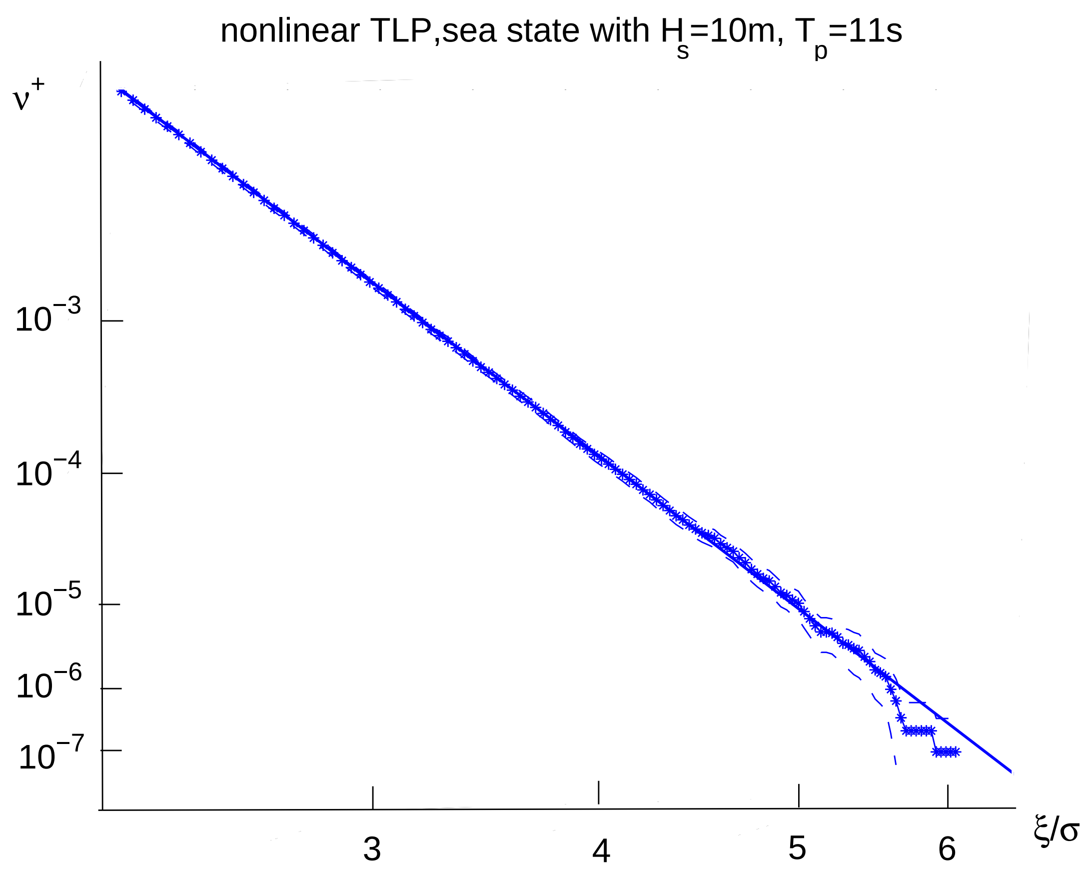
Crossing rates by Monte Carlo simulation (*) with 95% confidence bands (– –) and by saddle point integration (——) for the case of linear dynamics ($\tilde{c} = \tilde{\varepsilon} = 0$). Sea state with $H_s = 15$ m, $T_p = 17$ s.



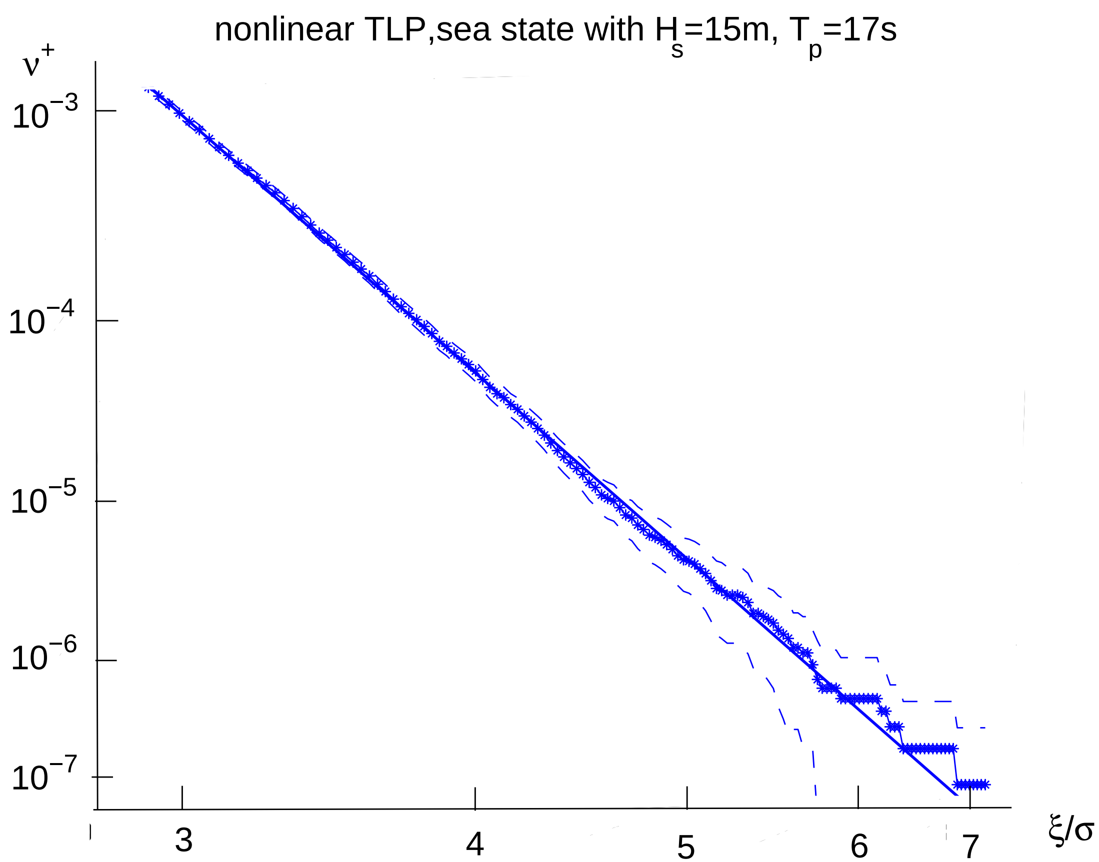
Numerical Examples - TLP

Crossing rates by Monte Carlo simulation (*) with 95% confidence bands (−−) for the case of nonlinear dynamics, $q = 0.2$, $b = 5.8\,\sigma_Z$. Sea state with $H_s = 10$ m, $T_p = 11$ s.



Numerical Examples - TLP

Crossing rates by Monte Carlo simulation (*) with 95% confidence bands (– –) for the case of nonlinear dynamics, $q = 0.2$, $b = 2.9\,\sigma_Z$. Sea state with $H_s = 15$ m, $T_p = 17$ s.



Conclusions

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Conclusions

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- Optimized linear fit and extrapolation on a double logarithmic scale gives accurate predictions of the mean upcrossing rate and thus extreme response statistics.
- The CPU time is in all examples tractable, and it is reduced by a factor of ≥ 100, compared to straight-forward Monte Carlo simulations down to the same extreme value levels.