Modelling and Identification of Radiation-force Models for Ships and Marine Structures

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My work at CeSOS

2004-2007
• Ship roll stabilisation
• System identification of manoeuvring models
• System identification of seakeeping models (rigid and flexible structures)
• Dynamic positioning with constrained control
• Control allocation for ride control of high-speed vessels
• Adaptive wave filtering
• Parametric roll (modeling and observer design)

2007-2012
• Risk assessment of autonomous systems (aerospace)
• Fault-tolerant systems
• Energy-based control vehicle dynamics
Motivation

Hull geometry and loading condition

Hydrodynamic Code → Identification → Cummins Equation

Non-parametric models: frequency response functions

Experiments & CFD → Model with Viscous Correction

Parametric fluid memory model
Non-parametric seakeeping models

System identification of parametric fluid-memory models

MSS FDI toolbox
Non-parametric seakeeping Models
RB Dynamics & Hydrodynamic forces

Excitation loads  Radiation loads  Total loads

\[ \mathbf{T} = \mathbf{T}_{\text{rad}} + \mathbf{T}_{\text{res}} + \mathbf{T}_{\text{exc}} \]

\[ \dot{\xi} = \mathbf{J}(\xi) \nu \]

\[ M_{RB} \dot{\nu} + C_{RB}(\nu) \nu = \mathbf{T} \]
Cummins’s Equation (1961)

Approximation RB dynamics:

\[ \dot{\xi} = J(\xi)\nu \]

\[ M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau \]

\[ \tau = \tau_{rad} + \tau_{res} + \tau_{exc} \]

Radiation forces:

\[ \tau_{rad} = -A_\infty \ddot{\xi} - \int_0^t K(t - t')\dot{\xi}(t') \, dt' \]

Cummins’s Equation:

\[ (M + A_\infty)\ddot{x} + \int_0^t K(t - t')\dot{x}(t') \, dt' + Gx = \tau_{exc} \]
Frequency-domain Model

Radiation forces:

\[ \tau_{rad}(j\omega) = \omega^2 A(\omega)\xi(j\omega) - j\omega B(\omega)\xi(j\omega) \]

Frequency-domain non-parametric model:

\[ (-\omega^2 [M + A(\omega)] + j\omega B(\omega) + G)\ddot{\xi}(j\omega) = \tau_{exc}(j\omega) \]

Abuse of notation:

\[ [M + A(\omega)]\dddot{\xi} + B(\omega)\ddot{\xi} + G\ddot{\xi} = \tau_{exc} \]
Summary non-parametric models

• Time domain:

\[(M + A_\infty)\ddot{\xi} + \int_0^t K(t - t')\dot{\xi}(t') \, dt' + G\xi = \tau_{exc}\]

• Frequency domain:

\[-\omega^2[M + A(\omega)] + j\omega B(\omega) + G)\xi(j\omega) = \tau_{exc}(j\omega)\]
Ogilvie’s Relations (1964)

• Frequency-dependent added mass and potential damping:

\[
A(\omega) = A_\infty - \frac{1}{\omega} \int_0^\infty K(t) \sin(\omega t) \, dt,
\]

\[
B(\omega) = \int_0^\infty K(t) \cos(\omega t)
\]

• Retardation functions

\[
K(t) = \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t) \, d\omega
\]

\[
K(j\omega) = \int_0^\infty K(t)e^{-j\omega t} \, d\omega = B(\omega) + j\omega[A(\omega) - A_\infty]
\]
System Identification of parametric fluid-memory models
Replacing the convolution

\[(M + A_{\infty}) \ddot{\xi} + \int_0^t K(t - t') \dot{\xi}(t') \, dt' + G\xi = \tau_{exc}\]

\[\mu = \int_0^t K(t - t') \dot{\xi}(t') \, dt' \equiv \dot{x} = \hat{A} x + \hat{B} \dot{\xi}, \quad \hat{\mu} = \hat{C} x + \hat{D} \dot{\xi},\]

Due to Markovian properties of the state-space model, significant gains in simulation speed can be obtained.

The parametric model is also convenient for analysis of stability and for control and estimation.
Replacing the convolution

- Time domain identification:

\[
\begin{align*}
\text{Data} & \quad B(\omega) \quad \longrightarrow \quad K(t) \\
\text{Mapping} & \quad \longrightarrow \quad \hat{K}(s) \quad \longrightarrow \\
\text{Indentification} & \quad \longrightarrow \quad \hat{A} \quad \hat{B} \\
\end{align*}
\]

- Frequency-domain identification:

\[
\begin{align*}
\text{Data} \quad B(\omega), \ A(\omega), \ \left[ A_\infty \right] \\
\text{Mapping} \quad K(j\omega) \quad \longrightarrow \quad \hat{K}(s) \\
\text{Indentification} \quad \longrightarrow \quad \hat{A} \quad \hat{B} \\
\text{Mapping} \quad \longrightarrow \quad \hat{C} \quad \hat{D}
\end{align*}
\]
Properties – background information

The following properties derive from the hydrodynamics, and have implications on the parametric models:

\[
\hat{K}_{ik}(s) = \frac{P_{ik}(s)}{Q_{ik}(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + \ldots + p_0}{s^n + q_{n-1} s^{n-1} + \ldots + q_0}
\]

<table>
<thead>
<tr>
<th>Property</th>
<th>Implication on Parametric Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (\lim_{\omega \to 0} K(j\omega) = 0)</td>
<td>There are zeros at (s = 0).</td>
</tr>
<tr>
<td>2) (\lim_{\omega \to \infty} K(j\omega) = 0)</td>
<td>Strictly proper.</td>
</tr>
<tr>
<td>3) (\lim_{t \to 0^+} K(t) \neq 0)</td>
<td>Relative degree 1.</td>
</tr>
<tr>
<td>4) (\lim_{t \to \infty} K(t) = 0)</td>
<td>Input-output stable.</td>
</tr>
<tr>
<td>5) The mapping (\dot{\xi} \mapsto \mu) is Passive</td>
<td>(K(j\omega)) is positive real.</td>
</tr>
</tbody>
</table>

Consistency and rationality requires us to use all this information in the identification problem.
System Identification

System Identification = Model structure selection + Parameter estimation

Time-domain identification:
- LS-fitting of the retardation function (Yu & Falness 1998)
- Realisation theory (Kristiansen & Egeland 2003)

Frequency-domain identification:
- LS-fitting of the retardation frequency response (Jeffreys 1984), (Damaren 2000)
- LS fitting of added mass and damping (Soding 1982), (Xia et. al 1998), (Sutulo & Guedes-Soares 2006)
Distortion of the retardation function

\[ K(t) = \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t) \, d\omega \]
LS-fitting retardation function

\[
\hat{K}_{ik}(t, \theta) = \hat{C}_{ik}(\theta) \exp(\hat{A}_{ik}(\theta)t) \hat{B}_{ik}(\theta) + \hat{D}_{ik}(\theta)
\]

- Complicated Optimisation problem.
- The non-unique model structure.
- Order of and initial parameters not easy to infer.
- Makes no use of the prior knowledge.
Realisation Theory

Discrete-time approximation:

\[ x_{k+1} = \Phi x_k + \Gamma u_k \]
\[ y_k = C x_k + D u_k \]

\[ \Rightarrow K_k = C \Phi^{k-1} \Gamma + D \]

Steps:

1) Form a Hankel matrix with the impulse response samples.
2) Do a singular value decomposition (SVD).
3) Obtain the order from the number of non-zero singular values.
4) Obtain the model matrices via factoriaistion.
5) Convert the model to continuous time.

\[ \mathcal{H}_k = \begin{bmatrix} K_1 & K_2 & \ldots & K_k \\ K_2 & K_3 & \ldots & K_{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ K_k & K_{k+1} & \ldots & K_{2k-1} \end{bmatrix} \]

\[ \begin{align*}
\Phi &= \Sigma_1^{-1/2} \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} & U_{13} \end{bmatrix} \Sigma_1^{1/2} \\
\Gamma &= \Sigma_1^{-1/2} \mathbf{V}_{11}^* \\
C &= U_{11} \Sigma_1^{1/2} \\
D &= h(0),
\end{align*} \]

\[ \mathcal{H}_k = [U_1 U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} [\mathbf{V}_1^* \mathbf{V}_2^*] = U_1 \Sigma_1 \mathbf{V}_1^* \]
Realisation Theory

- Relatively easy to implement.
- Does not require initial parameter estimates.
- Allows order detection.

- Starts from a distorted impulse response.
- Requires conversion to continuous time (further distortion).
- Makes no use of prior information.
- In general poor model quality.
The distortion of the impulse response may affect the order selection.

The containership example suggests $K_{55}(s)$ order 2 to 5
Example containership DOF3,3

Reconstruction of damping and added mass from the parametric approximation:

\[
\hat{\mathbf{A}}(\omega) = \text{Im}\{\omega^{-1} \hat{\mathbf{K}}(j\omega)\} + \mathbf{A}_\infty \\
\hat{\mathbf{B}}(\omega) = \text{Re}\{\hat{\mathbf{K}}(j\omega)\},
\]
Frequency-domain identification

The \(i,k\) entry of \(K(s)\) can be approximated by a rational transfer function:

\[
\hat{K}(s, \theta) = \frac{P(s, \theta)}{Q(s, \theta)} = \frac{p_ms^m + p_{m-1}s^{m-1} + \ldots + p_0}{s^n + q_{n-1}s^{n-1} + \ldots + q_0}
\]

\[
\theta = [p_m, \ldots, p_0, q_{n-1}, \ldots, q_0]^T
\]

Then we can estimate the parameters via LS using the frequency response computed using the data generated by hydrodynamic code:

\[
\theta^* = \arg \min_{\theta} \sum_l w_l (\epsilon_l^* \epsilon_l) \quad \epsilon_l = K(j\omega_l) - \frac{P(j\omega_l, \theta)}{Q(j\omega_l, \theta)}
\]

\[
K(j\omega) = B(\omega) + j\omega[A(\omega) - A_\infty]
\]

This problem is non-linear in the parameters, but it can be linearised.
Prior knowledge and constraints

From the physics of the problem we know that the transfer functions have

1. Relative degree 1
2. \( H_{ik}(s)=0 \) for \( s=0 \)
3. Stable
4. Passive
5. Minimum order approximation is 2

\[
\hat{K}_{ik}^{min}(s) = \frac{p_0 s}{s^2 + q_1 s + q_0}
\]

Some of these properties can be enforced in the structure of the model and its parameters without complicating the optimisation.
By imposing constraints on the model structure and parameters, we obtain a model that satisfy all the properties of the retardation functions and have a better quality.
Example semisub

Data from marinecontrol.org
Surge – order 5

Convolution Model DoF 11

Potential Damping DoF 11

Phase K(jw) [deg]

Added Mass DoF 11

CeSOS Conference 2013 | 27-29 March
Heave – order 8

Convolution Model DoF 33

Potential Damping DoF 33

Added Mass DoF 33
Roll-sway – order 8

Couplings are not necessarily passive $B(w) < 0$
What are we doing estimation wise?

Can we relate our point estimates to characteristic of the posterior for the parameter vector?

Bayes-Laplace Theorem:

\[ p(\theta|DI) = \frac{p(D|\theta, I)p(\theta|I)}{p(D|I)} \]
What are we doing estimation wise?

\[ K(j\omega) = U(\omega) + jV(\omega) \iff \begin{bmatrix} U(\omega_l) \\ V(\omega_l) \end{bmatrix} = f(\omega_l, \theta) \]

Measurement model:

\[ D_l = f(\omega_l, \theta) + n_l \]

Assumption 1 - Gaussian uncertainty: \( n_l \sim \mathcal{N}(0, \Sigma) \)
Assumption 2 – i.i.d. uncertainties
Assumption 3 – diffuse prior for the parameters
What are we doing estimation wise?

Data: \( D = \{ D_1, D_2, \ldots, D_N \} \)

\[
\begin{bmatrix}
U(\omega_l) \\
V(\omega_l)
\end{bmatrix} = f(\omega_l, \theta)
\]

\[
D_l = f(\omega_l, \theta) + n_l
\]

The likelihood function (measurement model):

\[
p(D|\theta, I) \propto \prod_l \exp \left[ \frac{1}{2} (D_l - f(\omega_l, \theta))^T \Sigma^{-1} (D_l - f(\omega_l, \theta)) \right]
\]

\[
= \exp \left[ \frac{1}{2} \sum_l (D_l - f(\omega_l, \theta))^T \Sigma^{-1} (D_l - f(\omega_l, \theta)) \right]
\]
What are we doing estimation wise?

Our estimates based on NLS correspond to the maximum a posteriori (MAP) estimates:

\[
\theta_{NLS} \equiv \theta_{MAP} = \arg \min_\theta \sum_l (D_l - f(\omega_l, \theta))^T \Sigma^{-1} (D_l - f(\omega_l, \theta))
\]

under the following assumptions:

Assumption 1 - Gaussian uncertainty:
Assumption 2 – i.i.d. uncertainties
Assumption 3 – diffuse prior for the parameters
Extension to the 2D data

- Some Hydrodynamic codes use strip theory to compute hydrodynamic non-parametric models.

- The infinite frequency added mass is not available.

- This is a simple extension to the frequency-domain method.
Example FPSO

3D Visualization of the Wamit file: fpsqow.gdf
DOF 33 joint estimation
DOF 35 and 53

Convolution Frequency Response

\begin{align*}
|K(j\omega)| & = 170 \\
& \quad \text{Freq. [rad/s]} \in [10^{-2}, 10^{1}] \\
\end{align*}

\begin{align*}
\angle K(j\omega) & = 100 \\
& \quad \text{Freq. [rad/s]} \in [10^{-2}, 10^{1}] \\
\end{align*}

DoF 53

\begin{align*}
\text{Added Mass} & = 2 \times 10^{8} \\
& \quad \text{Frequency [rad/s]} \in [10^{-2}, 10^{1}] \\
\end{align*}

\begin{align*}
\text{Damping} & = 10 \times 10^{7} \\
& \quad \text{Frequency [rad/s]} \in [10^{-2}, 10^{1}] \\
\end{align*}
Estimated added mass

Infinite Frequency Added mass coefficient estimates:

<table>
<thead>
<tr>
<th>True Value</th>
<th>Identified</th>
<th>Rel. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{33} = 1.7283e8$</td>
<td>$\hat{A}_{33} = 1.731e8$</td>
<td>1.5 %</td>
</tr>
<tr>
<td>$A_{35} = -3.463e7$</td>
<td>$\hat{A}_{35} = -3.7179e7$</td>
<td>0.18%</td>
</tr>
<tr>
<td>$A_{55} = 3.9154e11$</td>
<td>$\hat{A}_{55} = 3.9293e11$</td>
<td>0.35%</td>
</tr>
</tbody>
</table>
MSS FDI toolbox

\[ \omega, A(\omega), B(\omega), A_\infty, \text{FDIOpt} \]

FDIRadMod.m

Krad(s), [A_\infty]

EditAB.m
Ident_retardation_FD.m
Ident_retardation_FDna.m

Demo_FDIRadMod_WA.m
Fit_siso_fresp.m
Demo_FDIRadMod_NA.m
Conclusion

- Parametric fluid-memory models are used in simulators and also in analysis and design of wave–energy converters.

- Several methods have been proposed to identify parametric models from data computed by hydrodynamic codes.

- The identification based on the frequency response of the retardation function have advantages over other methods:
  - No need to compute high or low frequency data
  - No need to guess initial parameters
  - Automatic model order selection
  - Incorporates prior knowledge
  - Simple parameter estimation problem.
References


Thank you