Modelling of flexible slender systems for real-time simulation and control applications

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Cable systems – new challenges

• A variety of marine applications involves cable mechanics
  – Design tools for such systems are widely available
  – Usually modelled by the finite element method

• Model-based control and system observers of cable systems are under development

• Traditional tools for design are not found suitable because
  – Real-time performance of accurate models is hard to achieve
  – Flexibility in numerical time domain solvers is needed due to other requirements in systems simulations
  – Fast initialization of the model is needed, and static solvers should preferably not be used for initialization
Challenges for real-time simulations

- Real-time simulations of mechanical systems require that bottlenecks in model implementations are removed
  - Computational hardware issues
  - Re-formulation of models (our focus)

1. Remove stiffness properties of minor importance
   - Axial dynamics are very fast, but often of less interest
2. Avoid demanding computation operations
   - Matrix inversion (i.e. mass matrix) is probably one of the worst ($\sim O(n^3)$)

- Traditional methods (i.e. finite element method and finite difference schemes) suffers from one or both of these issues
Cable dynamics background

• Usually very high axial stiffness compared to the transversal stiffness
• Wave velocities
  – Transversal
  – Axial

• High stiffness = high wave velocity
• CFL condition

\[
\gamma = \max_i \frac{c_i h}{l_i}, \quad \gamma \leq 1
\]

Simulation time step
Length of element \( i \)
Highest wave velocity (transversal or axial)
Developing the EAC\textsuperscript{1} model (1)

- EAC model = Euler Angle Cable model
- Lagrangian coordinate \( s=\lbrack 0..L \rbrack \) from one end to the other
- Fixed-free boundary conditions

- Tangential vector given by Euler angles
- Kinematics:
  - Positions by integration of tangential vector from fixed point \( r_0 \)
  - Accelerations by time differentiation of positions

\[
\begin{align*}
t^i (\Theta^i) &= \begin{bmatrix} \cos \theta \cos \psi \\ \cos \theta \sin \psi \\ - \sin \theta \end{bmatrix} \\
\dot{r}^i (s, t, \Theta) &= r_0^i (t) + \int_0^s t^i (\Theta (\zeta, t)) (1 + \varepsilon (\zeta, t)) \, d\zeta \\
\ddot{r} (s, t) &= \ddot{r}_0 (t) + \int_0^s \frac{d^2}{dt^2} (t^i (\Theta) (1 + \varepsilon)) \, d\zeta
\end{align*}
\]

Developing the EAC model (2)

- Spatial discretization of integral
  - Written on matrix form

- d’Alembert’s principle to find equations of motion
  - No rotation inertia on any element
  - Axial dynamics may be removed
Longitudinal dynamics

- Strain dynamics for one element

- Axial natural frequency

- Well-conditioned system requires equal Courant numbers

- Common cable configurations: $c_i \gg c_t$

- EAC model’s separation of axial dynamics
  - Renders possible two spatial grids (for axial and longitudinal dynamics)
  - Ensures well-conditioned systems
  - Removal of axial dynamics possible

\[
\ddot{\varepsilon}_i + 2\zeta_0\omega_{0,i} \dot{\varepsilon}_i + \omega_0^2 \varepsilon_i = \frac{1}{m_i} T_i.
\]

\[
\omega_{0,i} = \sqrt{\frac{E_i A_i}{l_i m_i}} = \frac{1}{l_i} \sqrt{\frac{E_i A_i}{\rho_i}} = \frac{c_{l,i}}{l_i}
\]

Courant numbers

Well-conditioned

Longitudinal Transversal

\[
\gamma_i = \frac{c_i h}{l_i} \quad \gamma_t = \frac{c_t h}{l_t}
\]

$s=0$  \hspace{1cm}  $s=L$
Other issues for the EAC model

- A singularity is found
  - A method to solve this is proposed

- Boundary conditions
  - Free-free conditions developed
  - Fixed-fixed conditions developed
    - Based on a constraint formulation
    - No verification of these

- Matrix inversion is still needed
  - Full matrix (but symmetric)

\[
\begin{align*}
S^T_\Delta A^T M S_\Delta \ddot{q} + S^T_\Delta A^T M \left[ (C S_\Delta + 2 A \dot{S}_\Delta) \dot{q} + \dot{S}_\Delta J_t v \right] &= S^T_\Delta A^T F - S^T_\Delta A^T M R_0
\end{align*}
\]
Experimental verification of EAC model

• “Hanging chain” experiment
  – Rope: 2.75 meters (heavy – lead)
  – Small bottom weight
  – 30 sec. excitation (10 cm amplitude)
  – Measured positions of markers
  – Only gravitational loading (no hydrodynamics)

• Recorded positions of carriage
  – Used for input to simulations

• Compared experiments to simulations
  – 68 segments in numerical model
  – Axial elongation disregarded
Motion tracks seen from above

- Simulations shows larger responses than experiments
- Cable exhibits circular (whirling) response after approx 20 sec.

![Graph showing motion tracks with markers and directions for excitation.](image)
Amplitude responses

- Frequency-dependent responses compared
- Numerical simulations predicts too high amplitude
- Qualitatively similar results
- Simulations with circular excitations show the same trends
- High repeatability for experiments found (not shown)

Simulations (lines) and experimental results (marks) for marker 3, 7 and 10.
Conclusions on EAC model

• New model with separation (and possible removal) of axial dynamics
  – A concept for solving the real-time challenge of numerical stiffness is proposed

• Good coincidence between simulations and experiments
  – Demanding experiments (small amount of energy dissipation)
  – Deviation may be explained by inaccuracies in numerics and experiments

• Extensive use of matrix inversion – bottleneck for real-time simulations
  – Further studies on new model developments needed
Developing the RBC model (1)

- RBC model = Rigid bar cable model
- Based on the motion of rigid bars
- Generalized coordinates:
  - Position vector $r$
  - Bar direction vector $b$

- Translational motion
  - Newton’s second law

- Rotational motion
  - $b^x$ has deficient rank, more equations needed

- Introduce quadratic constraints
  - Time differentiation twice
  - Merge equations into one
  - Need some stabilization $u_L$, which is axial dynamics
Developing the RBC model (2)

• Traditional stabilization of linear constraints is not well suited
  – Feedback linearization strategy proposed

• The constraint controls axial dynamics
  – If fixed bar length, apply set-point control to nominal bar length ($\varepsilon=0$)

• If axial strain is included:
  – Find internal forces
  – Use material law to find expected bar length
  – Apply tracking control to expected bar length

• Bandwidth of axial dynamics can be tuned
  – Physical $\omega_{0,i} = \sqrt{E_iA_i/l_i m_i}$
  – Or user-defined based on minimum time step

Strain
$$\varepsilon = b^T \frac{b}{||b||} - L$$

Strain dynamics
$$\ddot{\varepsilon} = -K_{pL} \varepsilon - K_{dL} \dot{\varepsilon}.$$ 

Choose control parameters

$$K_{pL} = \omega_{nL}^2,$$
$$K_{dL} = 2\zeta_{nL}\omega_{nL}.$$ 

Axial dynamics with defined bandwidth

$$\dddot{\varepsilon} + 2\zeta_{nL}\omega_{nL} \ddot{\varepsilon} + \omega_{nL}^2 \varepsilon = 0.$$
Developing the RBC model (3)

- Introduces linear constraints:
  - Prescribed node positions / velocity
  - Interconnection of bars

- Applies traditional Baumgarte stabilization
  - May tune control gains
  - Trade-off with bandwidth and accuracy
  - The critical time steps dependent on chosen bandwidth

- This system is repeated for arbitrary number of bars
  - Equations of motion on matrix form

\[
\begin{align*}
  c_n &= r - \frac{1}{2} b - n_1 = 0, \\
  c_c &= r_1 + \frac{1}{2} b_1 + \frac{1}{2} b_2 - r_2 = 0, \\
  \ddot{c} + K_d \dot{c} + K_p c &= 0
\end{align*}
\]
Other issues for the RBC model

- Rotational inertia for each bar included
  - Different from EAC model
- Potential minor problem with length constraint
  - Solution proposed
- Two solutions for fixed-fixed boundary conditions proposed
  - Not verified
- No use of trigonometric functions (or rotation matrices)

- No matrix inversions needed during simulation

- Control gains for stabilization of constraints chosen by user
  - Controls minimum time step (Nyquist / Shannon)
  - Solves stiffness problem
Verification of RBC model - motions

• Compared to analytical model of double pendulum
  – Comparison of forces and motion
• Parameters:
  – Bar lengths: 3 meters
  – Bar masses: 1 kg
• Adjusted constraint control gains

Low gains: $\omega_n=70$ rad/s, $\zeta=1$

High gains: $\omega_n=400$ rad/s, $\zeta=1$
RMS analysis of motion

- RMS value integrated over simulation time

- Rapid decrease in error when control gains higher than 200 rad/s

- Computation time steady increase with control gains

- Real axial stiffness (assume steel bars) much higher

- Significant savings in computational time possible
Analysis of constraints

- Nodal controller and interconnection controller very small deviations
- Improved length controller performance with increased gains

Feedback stabilisation (solid) and Baumgarte stabilization (dashed).
Analysis of forces

- More accurate force estimation for higher gains
- Comparison with FEM (Abaqus™)
  - Assume steel bars
  - Very stiff system (axial)
  - Small time steps
  - FEM indicates very high peaks in internal forces

High gains:
\[ \omega_n = 400 \text{ rad/s}, \zeta = 1 \]

Low gains:
\[ \omega_n = 70 \text{ rad/s}, \zeta = 1 \]
RMS analysis of forces

- RMS analysis shows smaller deviations for higher gains
- Rapid decrease in error when control gains more than 100 rad/s
- Same trends as for motion analyses
Conclusions on RBC model

• New model with separation (but no removal) of axial dynamics

• Constraints must be stabilized
  – Prescribed end nodes
  – Interconnected bars
  – Bar lengths

• Constraint stabilization gains
  – Determines numerical dynamics
  – Solves stiffness problem for real-time performance

• Good coincidence with analytical system used for verification

• RBC model seems to be a good alternative for real-time simulations
Conclusions on the doctoral study

• Two new models for cable dynamics are proposed
  – Both have better real-time properties than traditional models
  – Both needs to be further verified for fixed-fixed boundary conditions

• The initial application – fish trawling – may benefit from these results, i.e.
  – Development of observers based on the real-time cable models
  – Development of control systems based on the maneuvering concept

• Current status:
  – The RBC model is enhanced and implemented in commercial software

Thank you for your kind attention!
References

