Frequency Detuning of Parametric Roll
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Grand Voyager, 2005

Parametric roll resonance and what it *might* look like on the inside
APL China, 1998

- Post-Panamax C11-class
- Route: Kaohsiung - Seattle
- Containers: 1/3 lost, 1/3 damaged
- Estimated damage: US $100M

www.cargolaw.com
Parametric Roll Resonance

Resonance Phenomenon

- Differential equations with time-varying periodic coeff.
- No extern. excitation needed
- Small parametric excitation may lead to large response

Parametric Roll Resonance

- Autoparametric resonance
- Container ships, cruise ships, and fishing vessels

\[ m\ddot{\phi} + d\dot{\phi} + k(t)\phi = \tau \]
\[ k = k_0 + k_t \cos(\omega_e t) \]

Parametric resonance when

- \( \omega_e \approx 2\sqrt{k_0/m} = \omega_\phi \)
- Wave length \( \approx \) Ship length
- Sufficient wave height
- Low roll damping

Nayfeh and Mook: *Nonlinear Oscillations*, Wiley, 2004
Effect of Waves

Wave trough amidships

Large water plane area

Large $\tau$

Wave crest amidships

Small water plane area

Small $\tau$, large $h$

Wave trough amidships

Large water plane area

Large $\tau$

Avoid Parametric Roll

Reducing the probability

— Change the ship design
  • Additional damping to dissipate roll energy
  • Modify the shape of the ship (hull form)
— Improved navigation (avoid conditions)

Lessening the impact

— Opposing roll moment
  • Fins
  • Fluid tanks
  • Moving mass
  • Rudder deflection
— Violate the frequency cond.:

\[ \omega_e \approx 2 \sqrt{\frac{k_0}{m}} = \omega_\phi \]

Frequency Detuning Control

— Change encounter frequency:

\[ \omega_e (u, \psi, \omega_0, \beta_n^w) = \omega_0 - ku \cos (\beta_n^w - \psi) \]
— Change heading and/or speed

\( \omega_e \): Encounter frequency
\( u \): Speed
\( \psi \): Heading angle
\( \omega_0 \): Modal wave freq.
\( k \): Wave number
\( \beta_n^w \): Encounter angle
Modelling of Parametric Roll

- Ship panel model
- Pressure over instantaneous submerged hull
- First-order wave effects
- Consider heave, pitch, and roll motions
- Quasi-steady approach for heave and pitch
- Identification of the hydrostatic/hydrodynamical model coefficients
- Functional expressions of the coefficients
6-DOF Ship Model

\[
\dot{\eta} = J(\Theta) \nu \\
M \dot{\nu} + D(\nu) \nu + C(\nu) \nu + k(\eta, t) = \tau_c + \tau_e
\]

**Main characteristics**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>281 m</td>
</tr>
<tr>
<td>Breadth</td>
<td>32.3 m</td>
</tr>
<tr>
<td>Draft</td>
<td>11.8 m</td>
</tr>
<tr>
<td>Displacement</td>
<td>76470 t</td>
</tr>
</tbody>
</table>

**Generalized position vector:** \( \eta = \begin{bmatrix} p^n^\top, \Theta^\top \end{bmatrix}^\top \in \mathbb{R}^6 \)

**Generalized velocity vector:** \( \nu = \begin{bmatrix} v^b^\top, \omega^b^\top \end{bmatrix}^\top \in \mathbb{R}^6 \)

**Transformation matrix:** \( J \in \mathbb{R}^{6 \times 6} \)

**Rigid-body inertia and added mass:** \( M \in \mathbb{R}^{6 \times 6} \)

**Damping matrix:** \( D \in \mathbb{R}^{6 \times 6} \)

**Coriolis/centripetal matrix:** \( C \in \mathbb{R}^{6 \times 6} \)

**Control and environmental forces:** \( \tau_c, \tau_e \in \mathbb{R}^{6} \)

**Gravity and pressure forces:** \( k(\eta, t) = k_g(\eta) + k_p(\eta, t) \in \mathbb{R}^{6} \)


www.cesos.ntnu.no Breu, Frequency Detuning of Parametric Roll
Pressure Forces

Assumptions

— Ship can be split into panels, each parametrized by \((a, b)\)
— Known pressure field, unchanged by the passage of ship
— Integrate instantaneous pressure over each panel \(i\) of the instantaneously submerged hull \(S_{w,i}\) to give force
  \[
  f_p = \sum_i f_i = \int_{S_{w,i}} \Psi_i(a, b) n(a, b) \, da \, db
  \]
— Parameters \(a\) and \(b\) are functions of \(\eta\) and \(t\)
1-DOF Roll Model

Assumptions

— Waves regular and sinusoidal
  \[ \zeta = \zeta_0 \cos (\omega_0 t - k_w x^n) \]

— Waves unaffected by passage of ship

— Encounter frequency approximated by
  \[ \omega_e = \frac{d}{dt} \left( \omega_0 t - k_w \int u^n dt \right) \approx \omega_0 - k_w \cos (\psi) u \]

— Roll coupled to heave and pitch

— Quasi-steady approach yields explicit solutions for heave/pitch
  \[
  \begin{bmatrix}
  z(t) \\
  \theta(t)
  \end{bmatrix} =
  \begin{bmatrix}
  \tilde{a}_z \zeta_0 \cos \left( \int_{t_0}^{t} \omega_e (\tau) d\tau + \tilde{\alpha}_z \right) \\
  \tilde{a}_\theta \zeta_0 \cos \left( \int_{t_0}^{t} \omega_e (\tau) d\tau + \tilde{\alpha}_\theta \right)
  \end{bmatrix}
  \]

**Signs and Definitions**

- \( \zeta \): Wave elevation
- \( \omega_0 \): Modal wave frequency
- \( k_w \): Wave number
- \( x^n \): Position
- \( \psi \): Heading angle
- \( u \): Forward speed

**Equation**

\[
\ddot{\phi} + d_{44} \dot{\phi} + k_{44} \phi + k_{\phi^3} + k_{z\phi} z \phi + k_{\phi\theta} \phi \theta
\]

1-DOF Roll Model (con’t)

\[ m_{44} \ddot{\phi} + d_{44} \dot{\phi} + \left[ \kappa_1 + \kappa_2 \cos \left( \int_{t_0}^{t} \omega_e dt + \kappa_3 \right) \right] \phi + \kappa_4 \phi^3 = \kappa_5 \sin \left( \int_{t_0}^{t} \omega_e dt + \kappa_6 \right) \]

Hydrostatic Coefficients

- Linear restoring moment: \( \kappa_1 \)
- Cubic restoring moment: \( \kappa_4 \)
  - Assumed to be constant
  - Identified by free decay test in calm water (simulations of 6-DOF model without waves)
  - Least square curve fitting

Hydrodynamical Coefficients

- Coupling roll – heave/pitch: \( \kappa_2, \kappa_3 \)
- External wave forcing: \( \kappa_5, \kappa_6 \)
  - \( \kappa_i = \kappa_i (u, \psi), i \in \{2, 3, 4, 5\} \)
  - \( \kappa_5 \approx 0 \) for head seas
  - Identified from simulations of the 6-DOF model
  - Least square curve fitting
Model Verification

### 6-DOF Ship Model

\[
\dot{\eta} = J(\Theta) \nu
\]

\[
M\ddot{\nu} + D(\nu) \nu + C(\nu) \nu + k(\eta, t) = \tau_c + \tau_e
\]

### 1-DOF Roll Model

\[
m_{44}\dddot{\phi} + d_{44}\ddot{\phi} + \left[\kappa_1 + \kappa_2 \cos\left(\int_{t_0}^{t} \omega_e dt + \kappa_3\right)\right] \phi + \kappa_4 \phi^3 = \kappa_5 \sin\left(\int_{t_0}^{t} \omega_e dt + \kappa_6\right)
\]

— Valid for constant and time-varying speed and heading angle

- Identical Mathieu-type equation for constant \(\omega_e\)
- Functional expressions for the hydrodynamical coefficients
- Analytical model

- Suitable for control purposes
Maximum Roll Angle

6-DOF Model

1-DOF Roll Model

1-DOF/Functional Expr.
Time-Varying Speed

**Frequency Detuning Control**

— Change encounter frequency:

\[ \omega_e (u, \psi, \omega_0, \beta_{nw}) = \omega_0 - ku \cos (\beta_{nw} - \psi) \]

— Change heading and/or speed

\[ \omega_e, u, \psi, \omega_0, k, \beta_{nw} \]

\( \omega_e \): Encounter frequency
\( u \): Speed
\( \psi \): Heading angle
\( \omega_0 \): Modal wave freq.
\( k \): Wave number
\( \beta_{nw} \): Encounter angle

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www.cesos.ntnu.no

Breu, Frequency Detuning of Parametric Roll
## Frequency Detuning Control Approaches

<table>
<thead>
<tr>
<th><strong>Model Predictive Control</strong></th>
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<tr>
<td>— Model-based control method</td>
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<tr>
<td>— Prediction to find an optimal control</td>
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<tr>
<th><strong>Extremum Seeking Control</strong></th>
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<tr>
<td>— Non-model-based adaptive control method</td>
</tr>
<tr>
<td>— Iterative control loop to find optimal control</td>
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<th><strong>$L_1$ Adaptive Control</strong></th>
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<td>— Guaranteed robustness with fast adaptation (decoupling)</td>
</tr>
<tr>
<td>— Low-pass filtered parametric estimate</td>
</tr>
</tbody>
</table>

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Hovakimyan and Cao: *$L_1$ Adaptive Control Theory*, SIAM, 2010
Model Predictive Control

Past inputs and outputs → 3-DOF Ship Model

Future inputs

Optimizer

Objective function

Constraints

Future errors

Predicted outputs

Reference trajectory

Results MPC

Extremum Seeking & Param. Resonance

Breu and Fossen: $\mathcal{L}_1$ Adaptive and Extremum Seeking Control Applied to Roll Parametric Resonance in Ships, Proc. 9th IEEE International Conference on Control and Automation, 2011
L₁ Adaptive Control—Overview

System
\[ \dot{x} = A_m x + b \left( \omega u + \theta^\top x + \sigma \right) \]
\[ y = c^\top x \]

State Predictor
\[ \dot{x} = A_m \hat{x} + b \left( \hat{\omega} u + \hat{\theta}^\top x + \hat{\sigma} \right) \]
\[ \hat{y} = c^\top \hat{x} \]

Adaptation Laws
\[ \dot{\hat{\omega}} = -\Gamma \text{Proj} \left( \hat{\omega}, -\hat{x}^\top Pb \right) \]
\[ \dot{\hat{\sigma}} = -\Gamma \text{Proj} \left( \hat{\sigma}, -\hat{x}^\top Pb \right) \]
\[ \dot{\hat{\theta}} = -\Gamma \text{Proj} \left( \hat{\theta}, -\hat{x}^\top Pb \right) \]

BIBS stable reference system:
\[ \dot{x}_r = A_m x_r + b \left( \omega u_r + \theta^\top x_r + \sigma \right) \]
\[ u_r = C(s) \omega^{-1} \left( k_g r(s) - \eta_r(s) \right) \]
\[ y_r = c^\top x_r \]

\[ \|\hat{x}\|_\infty \leq \sqrt{\frac{\theta_m}{\lambda_{\min}(P) \Gamma}} \]
\[ \|x_{\text{ref}} - x\|_\infty \leq \frac{\gamma_1}{\sqrt{\Gamma}} \]
\[ \|u_{\text{ref}} - u\|_\infty \leq \frac{\gamma_2}{\sqrt{\Gamma}} \]

Hovakimyan and Cao: L₁ Adaptive Control Theory, SIAM, 2010
Results Extremum Seeking & $\mathcal{L}_1$ Adaptive Control

Breu and Fossen: $\mathcal{L}_1$ Adaptive and Extremum Seeking Control Applied to Roll Parametric Resonance in Ships, Proc. 9th IEEE International Conference on Control and Automation, 2011
Estimation

**Challenge:** Frequency detuning control assumes knowledge of wave encounter frequency and modal wave frequency.

**Extended Kalman Filter**
- Model-based estimation
- Optimal in the sense of minimum variance

**Linear Frequency Estimator**
- Signal-based estimation
- Globally convergent

**Frequency Estimation for Irregular Seas**
- Empirical mode decomposition, intrinsic mode function
- Ongoing research
Conclusions

— Nonlinear resonance phenomenon
  - Small parametric excitation may lead to large roll angles
  - Dangerous for container ships, cruise ships, and fishing vessels

— The steps to solve the problem
  - Understand the phenomenon
  - Mathematical modelling
  - Control approaches
  - Parameter estimation
Thank you for your attention
Further Reading I


– (2013). A $\mathcal{H}$-Exponential Stable Nonlinear Observer for the Wave Encounter Frequency. In: *Submitted to the 9th IFAC Conference on Control Applications in Marine Systems*. 
Further Reading II


Further Reading III


Further Reading IV


