Integrated analysis of hydraulic PTOs in WECs

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Content

• Introduction
• Model description of wave energy converter (WEC) with hydraulic power take-off (PTO)
• Bond graph representations of pipelines
• Bond graph system modelling of the WEC
• Simulation results
• Conclusions
• Publications
WEC introduction

Wear
Fatigue

Stochastic wave excitation

Catastrophic failure
Fatigue
The studied wave energy converter can be characterized by several subsystems and each of which is based on basic physical laws.

The electric generator used here is simplified as a dissipative element.
Model description — wave-buoy

Assumptions and simplifications:

- Linear hydrodynamic theory
- Heave motion only

\[
(M + m_\infty) \ddot{X} + \int_0^t K(t - \tau) \dot{X}(\tau) d\tau + K_{\text{hstatic}} X = F_{\text{exc}}(t) + F_{\text{ext}}(X, \dot{X}, t)
\]

- Mass
- Acceleration
- Velocity
- Position
- External force

- Radiation force
- Hydrostatic stiffness
- Excitation force

wave-buoy interactions
Model description — wave-buoy

The governing equation of the buoy includes a convolution integral term which can be approximated by a state space model:

\[
\dot{z}(t) = Az(t) + B\dot{X}(t)
\]

\[
\int_{-\infty}^{t} K(t - \tau) \dot{X}(\tau) d\tau \approx Cz(t)
\]

The matrix coefficients $A$, $B$ and $C$ of the state space equations can be calculated by using Matlab function `imp2ss`. 
Model description — pump

Applying the mass balance law to the respective chambers gives state space equations for the chamber pressures:

\[ \dot{P}_A = \frac{\beta}{A_p (L - X)} (Q_A - Q_{li} + A_p \dot{X}) \]

\[ \dot{P}_B = \frac{\beta}{A_r (L + X)} (Q_B + Q_{li} - Q_{le} - A_r \dot{X}) \]

Assuming that the buoy and piston are rigidly connected by the rod, the external force defined in the buoy motion equation can be written as:

\[ F_{\text{ext}} (X, \dot{X}, t) = A_r P_B - A_p P_A - F_f (t) - F_{\text{end}} (t) \]
Model description — motor

Practical model:

\[
\dot{P}_1 = \frac{\beta}{V_{pf}} (Q_{in} - \omega D - Q_{loss})
\]

\[
\dot{P}_2 = \frac{\beta}{V_{pb}} (\omega D - Q_{out} - Q_{loss})
\]

\[
J_m \ddot{\theta} = (P_1 - P_2) D - B_m \dot{\theta} - T_L
\]

Here \( D \) is the volume displacement defined as:

\[
D = NA_p R \tan \alpha / \pi
\]

Swash plate axial piston motor
Model description — check valves

Check valves are used to control the direction of the fluid flow. They can be considered as resistances that cause pressure drops when the fluid flows across them. They can be modelled by using Bernoulli’s energy equation by combining a position dependent function:

\[ Q = C_d A_{val} \Omega(t) \sqrt{\frac{2}{\rho} |\Delta P| \text{sign}(\Delta P)} \]

\[ \Omega(t) = f(\zeta), \quad \text{with} \quad \zeta = \frac{x_{val}(t)}{x_{val,stop}}. \]

**Stroke position dependent function**

**Determined by valve type**

**Maximum displacement the stroke reaches**
Model description — check valves

Water hammer pressure waves can be created by check valves sudden closure or opening.

\[ \Delta P = \frac{\rho}{g} \Delta u \]

Returned period: \( T_r = \frac{2L}{a} \)

Valve closure time: \( t_c \)

Diagram showing a tank connected to a valve with symbols for fluid velocity, tank length, and pressure wave characteristics.

Legend:
- \( \Delta P/\rho g \Delta u \)
- time \((a/L)t\)
Model description — water hammer pressure

The magnitude of water hammer pressure amplitudes can be approximated by Joukowski’s equation for complete closure or opening of the valve.

Joukowski’s equation

\[ \Delta P = \rho a \Delta u \quad \text{for} \quad t_c \leq T_r \]

For partial valve actuations, time dependent pressure transient amplitudes can be calculated by solving the basic partial differential equations of pipelines with the dynamic valve boundaries.
Model description — pipelines

The components of the hydraulic power take-off are connected by the hydraulic lines. The fluid pressure changes propagate with a speed of sound. Relatively long lines may introduce:

- A time delay for the pressures at the upstream and downstream sides.
- Strong pressure pulsations (water hammer) during the transition from one steady state to another. This is mainly induced by:
  - Cyclic operation of check valves
  - Sudden pump shut off
  - Suddenly start or stop the system
  - Failure of the valves
Model description — pipelines

The model of the hydraulic pipeline is found from the mass and momentum balance and can be written as partial differential equations:

\[
\frac{\partial P(x,t)}{\partial t} + \frac{\rho a^2}{A} \frac{\partial Q(x,t)}{\partial x} = \frac{\rho a^2}{A} S_Q(x,t)
\]

\[
\frac{\partial Q(x,t)}{\partial t} + A \frac{\partial P(x,t)}{\partial x} = \frac{F(x,t)}{\rho}
\]

Basic assumptions:

- Pipe wall is rigid;
- The flow is laminar;
- The motion in radial direction is negligible;
- Thermodynamic effects are negligible

Fluid damping

\[
F_f(Q) = \rho B Q + \frac{1}{2} \rho B \int_0^t w(t-\tau) \frac{\partial Q(\tau)}{\partial t} d\tau
\]

1-D friction term

2-D friction term
Model description — pipelines

There are four possible sets of boundary conditions, corresponding to four input-output configurations, that lead to causal line models.

1. \([P_{up}, P_{down}]\) as input
   e.g. the line is connected to volumes at both sides

2. \([Q_{up}, Q_{down}]\) as input
   e.g. the line is connected to valves at both sides

3. \([P_{up}, Q_{down}]\) as input
   e.g. the line is connected to a valve at one port and a volume at the other port.

4. \([Q_{up}, P_{down}]\) as input
Using the modal approximation method, the pipeline dynamics can be characterised as a series of damped resonant modes. Each mode can be represented in a linear state space form.
This method begins by assuming that the pressure $P$ and flow rate $Q$ can be separated into a product of mode shapes of $x$ and a modal generate coordinate of time $t$.

$$P(x,t) = \sum_{i=1}^{\infty} H_i(x) \eta_i(t)$$

$$Q(x,t) = \sum_{i=1}^{\infty} G_i(x) \xi_i(t)$$

The mode shapes are given by the homogeneous solution of the pipeline equations. By using the orthogonality property of the mode shape, a set of decoupled ODEs can be obtained for each normal mode. The bond graph representations can then be obtained. The ODEs can be solved by numerical integration to find modal responses.
Model description — pipelines (RTF method)

The one-dimensional distributed transmission line model can be expressed by the four-pole equations that relate to different boundary conditions in the Laplace domain. The major obstacle to the distributed parameter model is that the terms of the transfer function are not in the form of a finite rational polynomial. The basic idea of RTF method is to represent each of the transcendental function as finite sum approximations of low-order polynomial forms.

\[
T(s) = \sum_i T_i(s)
\]

Transcendental transfer function
Polynomial transfer function
Bond graph models

A **bond graph** is:

- A graphical representation of physical dynamic system
- A multidisciplinary and unified approach
- Mainly used for modelling the systems in which power and energy interactions are important.

*Bond graph models* include nine basic elements. A graphical model can be constructed by using these elements for a system.
Bond graph models—pipelines (SOV method)

The bond graph models by using SOV are directly shown in the figures.

1. \([P_{up}, P_{down}]\) as input (by Karnopp)

2. \([Q_{up}, Q_{down}]\) as input (by Karnopp)

3. \([P_{up}, Q_{down}]\) as input (By Yang et al.)

4. \([Q_{up}, P_{down}]\) as input (By Yang et al.)
Bond graph models—pipelines (RTF method)

The input-output behaviour governed by each polynomial transfer function can be represented by using bond graph models. For different causalities, the suggested bond graph models are shown in the figures.

1. \([P_{up}, P_{down}]\) as input (by Yang et al.)
2. \([Q_{up}, Q_{down}]\) as input (by Yang et al.)
3. \([P_{up}, Q_{down}]\) as input (By Margolis)
4. \([Q_{up}, P_{down}]\) as input (By Margolis)
Simulation results—method comparison

3. Pressure pulsations at the upstream side of the pipeline with \([Q_{up}, Q_{down}]\) as input
Simulation results—friction effect

Pressure transient prediction with \([P_{\text{up}}, Q_{\text{down}}]\) as input

1-D friction term

\[ F_f (Q) = \rho B Q + \frac{1}{2} \rho B \int_0^t w(t-\tau) \frac{\partial Q}{\partial t} (\tau) d\tau \]

2-D friction term
WEC model — Interconnections of subsystems

A sketch diagram which shows the input-output behaviour within each component.
WEC model — bond graph representation
Simulation results

Under the sea state $H_{s,des} = 3.5 \text{ m}$, $T_e = 9.5 \text{ s}$

**Pressure in pump cylinder**

**Pressure in HP accumulator**

**Flow rate pumped into HP accumulator**

**Power available to pump and motor**
Conclusions

Developed an integrated dynamic model for a wave energy converter with hydraulic PTO using inter-linked models of the related subsystems.

Constructed bond graph models for transmission pipelines by using SOV and RTF for all the four possible input-output causalities.

Comparisons were made for the pipeline responses by using the SOV and RTF with different causalities in time domain. It shows that the transient properties can be preserved well for both types of methods.

Extended the pipeline bond graph models from 1-D friction model to 2-D friction model.
Relevant publications


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