Effect of nonlinear Froude-Krylov and restoring forces on a hinged multibody WEC

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How the WEC works

• N buoys hinged to a soft-moored central platform
• The energy is absorbed from the relative rotation between the bodies
• The diagonal mode of motion ensure a good efficiency without having to apply large control forces on a wave to wave timescale
Motivation

• Optimizing Power/displacement or similar cost indicators typically result in rather small WEC’s with large amplitude motion
• Large motion amplitude relative to characteristic length → Violation of linear theory
• CFD codes are too time consuming for long simulations and parameter variations

Aim:

Develop a simulation tool that captures the most important nonlinearities and are fast and accurate enough for design optimization and parameter variations.

Method:

Integrating the hydrostatic and undisturbed dynamic pressure over the time varying wetted surface under the undisturbed waves, while still relying on linear diffraction and radiation forces.

Previous work (i.e. [1]) has shown that this approach can describe non-linear phenomena.

Model test

- MARINTEK’s MCLab (1:30 scale)
- Power take-off by pneumatic cylinders
- Change damping characteristics by changing nozzle diameter and lever arm
- Considered a 3 body system since 2 out of 4 cylinders were fixed by static friction!
Key geometric parameters (full scale)

Displacement, center floater: $572 \text{ m}^3$
Displacement, 1 buoy: $98 \text{ m}^3$
Displacement, CF + 4 buoy’s: $963 \text{ m}^3$
Equation of motion – generalized coordinates

\[ M(q_4, q_5) \ddot{\Omega} + C(\Omega, q_4, q_5) \dot{\Omega} = Q \]

\[ V = J(q_4, q_5) \dot{\Omega} \]

<table>
<thead>
<tr>
<th>Generalized coordinates (displacements):</th>
<th>Independent velocities:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1 = \text{Collective surge} )</td>
<td>( \Omega_1 = \cos(q_3) \dot{q}_1 - \sin(q_3) \dot{q}_2 )</td>
</tr>
<tr>
<td>( q_2 = \text{Collective heave} )</td>
<td>( \Omega_2 = \sin(q_3) \dot{q}_1 + \cos(q_3) \dot{q}_2 )</td>
</tr>
<tr>
<td>( q_3 = \text{Collective pitch} )</td>
<td>( \Omega_3 = \dot{q}_3 )</td>
</tr>
<tr>
<td>( q_4 = \text{Hinge angle 1} )</td>
<td>( \Omega_4 = \dot{q}_4 )</td>
</tr>
<tr>
<td>( q_5 = \text{Hinge angle 2} )</td>
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Reference:
Rogne, Ø. Y. & Pedersen, E. Bond Graph Modeling of a Multibody Wave Energy Converter
*In Proceedings of the International Conference on Bond Graph Modeling (ICBGM), 2012*

(More general: 6 collective modes + N hinges)
Generalized forces

Nonlinear!

\[ Q = Q_{HS} + Q_{FK} + Q_D + Q_R - B_{PTO} \Omega - K_{moor} q + Q_{drag} + Q_{endstop} \]

Compare results with «traditional» model:

\[ Q_{HS} \approx -K_{HS} q \]
\[ Q_{FK} \approx Q_{FK,lin} \]
Non-linear Froude-Krylov and hydrostatic forces

Force and moment on body \( i \) in its own frame

\[
F_{HSFK,i}^{(i)} = \iint_{S_i(t)} p_u \left( r_s'^n \right) \begin{bmatrix} n_s^{(i)} \\ r_s^{(i)} \times n_s^{(i)} \end{bmatrix} ds - mg \begin{bmatrix} k^{(i)} \\ r_{CG,i}^{(i)} \times k^{(i)} \end{bmatrix}
\]

From undisturbed dynamic and hydrostatic pressure

Generalized force

\[
Q_{HSFK} = \mathbf{J} \begin{bmatrix} q_4 \\ q_5 \end{bmatrix}^T \begin{bmatrix} F_{HSFK,1}^{(1)} \\ \vdots \\ F_{HSFK,n}^{(n)} \end{bmatrix}
\]

From gravity

Non-linear relation to surface elevation because:
- Wetted surface change with time
- Gravity change direction with time (in body fixed frame)
- Kinematic transformation matrix change with time
Undisturbed pressure formulation

- Use linear (Airy) wave theory
- Neglect quadratic term in Bernoulli’s equation

\[
p_u = -\rho gz - \rho \frac{\partial \phi_0}{\partial t}
\]

\[
\frac{\partial \phi_0}{\partial t} = -g \sum_{i=1}^{n} A_i \frac{\cosh (k_i z' + k_i h)}{\cosh (k_i h)} \cos (\omega_i t - k_i x + \epsilon_i)
\]

\[
z' = \min(0, z)
\]

Simple continuation above mean water level
Accept non-zero relative pressure in water surface below mean water level!
Froude Krylov force vs. diffraction force

- Collective surge
- Collective heave
- Collective pitch
- Hinge torque, up-wave
- Hinge torque, down-wave

Graphs showing the comparison of Froude-Krylov force and diffraction force against excitation.
Wave radiation forces in time domain, 1

State space approximation of convolution integral:

\[ \dot{x} = \hat{A} x + \hat{B} \Omega \]
\[ Q_R = -\hat{C} x - A_\infty \dot{\Omega} \]

State space matrices from frequency domain curve fitting:

\[ K(j\omega) \rightarrow \hat{A}, \hat{B}, \hat{C} \]
\[ K(j\omega) = B(\omega) + j\omega (A(\omega) - A_\infty) \]

Generalized added mass and damping:

\[ A(\omega) = J_0^T A_W AMIT(\omega) J_0 \]
\[ B(\omega) = J_0^T B_W AMIT(\omega) J_0 \]

\[ J_0 = J(q_4 = 0, q_5 = 0) \]

Consistent with linear theory!
Wave radiation forces in time domain, 2

We choose poles and residues as free variables in the curve fitting. Poles shared between coupling terms.

\[ K(s) \approx \sum_{i=1}^{n} \frac{R_i}{s - p_i} \]

More common: use polynomial coefficients as free variables.

\[ K_{ij}(s) \approx \frac{b_{n-1}s^{n-1} + \cdots + b_1 s}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \]

Advantages:

- Suited for strongly frequency dependent transfer function (multibody systems)
- Enables «passivation» of slightly non-passive transfer function approximations

References:

Rogne, Ø. Y., Moan, T., Ersdal, S.
“Identification of passive state-space models of strongly frequency dependent wave radiation forces”
*Submitted to Ocean Engineering*, 2013.

Gustavsen, B., Semlyen, A.
Wave radiation forces in time domain, 3

- Curve fitting of added mass and damping (24 poles)
Overprediction

Linear: Between 15% and 61% (average 33%)
NonLin: Between -8.6% and 21% (average 9.5%)
Spectra of hinge angle (Hs=3.1 m, Tp=7.1 s)

Interaction between frequencies!
Cumulative distribution of max and min hinge angle

Max/Min → Largest and smallest angle between subsequent upcrossings of mean angle

End-stop spring!
Pitch – Spectrum and Max/Min distribution
Mean angular offset

Equilibrium position (Linearization position in BEM analysis)

Waves

Mean position (exaggerated)

- Should re-run BEM with a-priori information
- The mean hinge angle is a sensitive parameter!
Surge
(Hs=3.1 m, Tp=7.1 s)

Important for:
- Design of mooring system
- Design of power cable connection

Displacement spectrum

Cumulative distribution of min and max

\[ q_1, Surge \ [m] \]
Surge

- Same horizontal drag coeff. in all sea states
- Mean offset insensitive to drag coefficient
- WAMIT’s 2’nd order drift coefficients is not used!
Sensitivity to PTO damping

Possible to have increase in power and decrease in motion amplitude!
Animation (Hs=3.1 m, Tp=7.1 s)
Thank you!
References

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