Overview:
Research on Wave loading, Response and Marine Operations

*Hydroelastic Responses in Waves
--- Pontoon type
--- Semi-sub type
--- Analytical features

*Hydroelastic Responses in Various Conditions
--- Time-domain analysis
--- Towing condition
--- Assembling condition

• Risk Analysis of VLFS Mooring system
--- Wind force
--- Slowly varying wave drift force

--- Wavelengths are very small compared to horizontal size of typical VLFS
\[ \lambda / L = 1/50 - 1/100 \]

• Flexural rigidity is relatively small
Hydroelastic responses are more important than rigid-body motions

Conventional methods can not be applied and
Many studies were undertaken and developed
Hydroelastic Response in Waves

Representation of elastic motions

Mode-expansion method

Succession of vertical displacement of substructures

Mesh method
## Hydroelastic Response in Waves

### Representation of Elastic Motions

<table>
<thead>
<tr>
<th>Treatment of Hydrodynamic Forces</th>
<th>Mode-expansion method</th>
<th>Mesh method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integral equation method</td>
<td>Kashiwagi (1998)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Seto (1998)</td>
<td></td>
</tr>
</tbody>
</table>

### Modified Free Surface Condition

<table>
<thead>
<tr>
<th>Treatment of Hydrodynamic Forces</th>
<th>Mode-expansion method</th>
<th>Mesh method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green function method</td>
<td>Ohkusu &amp; Namba (1998)</td>
<td></td>
</tr>
<tr>
<td>Integral equation method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eigenfunction expansion-matching method</td>
<td>Kim &amp; Ertekin (1998)</td>
<td></td>
</tr>
</tbody>
</table>

---

For Semi-sub type VLFS

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Iijima et al. (1997)</td>
</tr>
<tr>
<td></td>
<td>Kashiwagi (1998b)</td>
</tr>
<tr>
<td></td>
<td>Murai &amp; Kagemoto (1999)</td>
</tr>
</tbody>
</table>

---

### Model Experiment in Wave Basin

![Model experiment in wave basin](image)
Hydroelastic Response in Waves

Analytical consideration of Characteristics of Hydroelastic Response

Equation of vertical displacement \( \zeta(x, t) \) of thin beam

\[
m \frac{\partial^2 \zeta}{\partial t^2} + EI \frac{\partial^4 \zeta}{\partial x^4} + \rho g \zeta = -i \omega \phi
\]

Body boundary condition

\[
\frac{\partial \phi}{\partial z} = \frac{\partial \zeta}{\partial t}
\]

Modified free surface condition

\[
(1 - \frac{m \omega^2}{\rho g} + \frac{EI}{\rho g} k^4)k \tanh kh = \frac{\omega^2}{g} \equiv K
\]

Water surface

\[
\frac{\partial \phi}{\partial z} = \frac{\omega^2}{g} \phi \equiv K \phi
\]

Modified dispersion relation

\[
\omega_0^2 = \left(\frac{\rho g}{m}\right) k_p^4 = \left(\frac{\rho g}{EI}\right)
\]

\[
1 - \left(\frac{\omega}{\omega_0}\right)^2 + \left(\frac{k}{k_p}\right)^4
\]

\[
k \tanh kh = \frac{\omega^2}{g} \equiv K
\]
Hydroelastic Response in Waves

Analytical consideration of Characteristics of Hydroelastic Response

Dispersion relation

\[
\frac{V_{p}}{k} = \frac{\omega}{k_{p}} = \sqrt{\frac{g}{k_0^2 + gk \tanh kh}}
\]

\[
V_{p} = \frac{\omega}{k_{0}} = \sqrt{\frac{g \tanh kh}{k_0}}
\]

\[
h \to 0 \quad V_{p} = \frac{1 + (k / k_p)^4}{\sqrt{1 + (gk / \omega_0^2)}} \times \sqrt{gh}
\]

\[
h \to \infty \quad V_{p} = \frac{1 + (k / k_p)^4}{\sqrt{1 + (gk / \omega_0^2)}} \times \sqrt{\frac{g}{k}}
\]
Hydroelastic Response in Waves
Analytical consideration of Characteristics of Hydroelastic Response

Snell’s law

\[
\frac{\sin \alpha}{\sin \beta} = \frac{c_a}{c_b} = \frac{1}{n}
\]

\(n = \) Refraction coefficient

VLFS is Prism ?? ??
Hydroelastic Response in Waves
Analytical consideration of Characteristics of Hydroelastic Response

\[ \alpha = 22.9 \text{ deg.} \]

Critical angle
\[ \alpha = 20.9 \text{ deg.} \]

\[ \alpha = 18.9 \text{ deg.} \]

Amplitude transmission coefficient of flexural waves

\[ \phi = \zeta_a \frac{i \omega \cosh (z + h)}{k \sinh kh} \cdot \exp[i(\omega t - kx)] \]

\[ p = \rho \zeta_a \frac{\omega^2 \cosh (z + h)}{k \sinh kh} \cdot \exp[i(\omega t - kx)] \]

\[ u = \zeta_a \omega \frac{\cosh (z + h)}{k \sinh kh} \cdot \exp[i(\omega t - kx)] \]

\[ p = \rho \frac{\omega}{k} u = \rho cu \]
Hydroelastic Response in Waves

Analytical consideration of Characteristics of Hydroelastic Response

Amplitude transmission coefficient of flexural waves

Mean pressure on the boundary

\[ \rho c_a(u_a + u_c) = \rho c_b u_b \]

Mean velocity perpendicular to boundary

\[ u_a \cos \alpha - u_c \cos \alpha = u_b \cos \beta \]

Relation of mean velocity and amplitude

\[ \zeta_a = \frac{h u}{c} \]

Amplitude transmission coefficient of flexural waves

\[ RAO = \frac{\xi_a}{\eta_a} = \frac{u_b c_a}{c_b u_a} = m \cdot U_T \]

\[ U_r = \frac{u_b}{u_a} = \frac{2m \cos \alpha}{\cos \alpha + m \cos \beta} \]

\[ m = \frac{c_a}{c_b} = \frac{1}{n} \]
Transmission coefficient and reflection coefficient of mean velocity

\[ U_T = \frac{u_b}{u_a} = \frac{2m \cos \alpha}{\cos \alpha + m \cos \beta} \quad U_R = \frac{u_c}{u_a} = \frac{\cos \alpha - m \cos \beta}{\cos \alpha + m \cos \beta} \]

Amplitude transmission coefficient of flexural waves

\[ RAO = \frac{\zeta_a}{\eta_a} = \frac{u_b}{c_b} \frac{c_a}{u_a} = m \cdot U_T \]

\[ \alpha = \beta = 0 \quad RAO = \frac{2m^2}{1 + m} \]

Directional Dependence of Longitudinal Bending Moment Response Amplitude
Hydroelastic Response in Various Conditions

Time-domain analysis method

--- irregular waves
--- airplane landing and take-off

Ohmatsu (1999)
Frequency-domain ---- Fourier Transform

Kashiwagi (2004)
Direct computation using Time-domain mode-expansion method
Hydroelastic Response in Various Conditions

Towing of unit structure of VLFS

Hara et al. (2004)
At-sea experiment of a Mega-Float unit

Hydroelastic Response in Various Conditions

Towing of unit structure of VLFS

Hara et al. (2004)
At-sea experiment of a Mega-Float unit

Watanabe & Utsunomiya (1998)
Wave response analysis of an elastic floating plate in a week current
Perturbation expansion of velocity potential and Green function with U
Hydroelastic Response in Various Conditions

Floating units are assembled to construct VLFS

Yago (2000)
Various connection conditions --- free, spring, pin-joint

Takaki (1998)

Quantitative Risk Analysis of VLFS Multiple Mooring Dolphins

Wind
Pressure drag + Friction drag

Waves
Linear wave force
Slowly varying wave drift force

Current
Quantitative Risk Analysis of VLFS Multiple Mooring Dolphins

\[
\begin{align*}
M_{ij} + m_{ij}(\infty) & \dot{X}(t) + F_{V}(\dot{X}) + \sum_{j=1}^{n} \int_{-\infty}^{t} \dot{x}_j(t) L_{ij}(t-\tau) d\tau + F_{M}(X, \dot{X}) \\
& = F_{\text{wind}}(t) + F_1(t) + F_2(t)
\end{align*}
\]

Wind Force

\[
S_{FF}(f) = \rho u_0^2 \int_{-\infty}^{\infty} C_{d_h}(f) C_{d_j}(f) U_i U_j \text{Re} \left[ R_{ij}(f) \right] \sqrt{S_i(f) S_j(f)} dA_i dA_j
\]

\[
R_{ij}(f) = \exp \left( -\frac{k_1 f |y_i - y_j|}{\sqrt{U_i U_j}} \right) \exp \left( i \frac{k_2 f (x_i - x_j)}{\sqrt{U_i U_j}} \right)
\]
Quantitative Risk Analysis of VLFS Multiple Mooring Dolphins

**Slowly Varying Wave Drift Force**

\[
F_2(t) = \rho \int_{SB} dS \Phi^{(1)} \nabla w^{(1)} + \frac{1}{2} \rho g \int_{C} dC (w^{(1)})^2 n^{(0)} - \frac{1}{2} \rho g \int_{C} dC (\xi^{(1)})^2 n^{(0)}
\]

\[
D \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w^{(1)} = -\rho g w^{(1)} - \rho \Phi^{(1)} \quad z = 0
\]

\[
F_2(t) = -D \int_{SB} dS \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 w^{(1)} - \frac{1}{2} \rho g \int_{C} dC (\xi^{(1)})^2 n^{(0)}
\]

\[
F_2(t) = -D(1-\nu) \int_{C} dC \left( \frac{\partial^2 w^{(1)}}{\partial n \partial \tau} \right)^2 n^{(0)} - \frac{D}{2} (1-\nu^2) \int_{C} dC \left( \frac{\partial^2 w^{(1)}}{\partial \tau^2} \right)^2 n^{(0)}
\]

\[
- \frac{1}{2} \rho g \int_{C} dC (\xi^{(1)})^2 n^{(0)}
\]
Quantitative Risk Analysis of VLFS Multiple Mooring Dolphins

Slowly Varying Wave Drift Force

\[ F_z(t) = -\frac{1}{2} \rho g \left\{ \sum_{i} \sum_{j} \eta_{ij} \left( \zeta^{(i)} \right)^* \right\} n \]

\[ \eta_j(x; t) = \text{Re} \left[ \sum_{i} \sum_{j} \eta_{ij} e^{i(k_j x - \omega_j t)} \right] \]

\[ \zeta(x; t) = \text{Re} \left[ \sum_{i} \sum_{j} \eta_{ij} \zeta_{ij}(x) e^{-i\omega_j t} \right] \]

\[ F_z(t) = \text{Re} \left[ \sum_{i} \sum_{j} \sum_{k} \sum_{l} F^{-i j k l} e^{-i(\omega_k - \omega_j) t} \right] \]

\[ F^{-i j k l} = -\frac{1}{4} \rho g \eta_{ij} \eta_{kl} \int_{C} \xi_{ij} \xi_{kl}^* \eta \]

Simple method

\[ f_y = \rho g \left\{ \sum_{i} \sum_{j} \eta_{ij} S_i \sin \theta_j \cos(\omega_j t + \epsilon_{ij} - K_i x \cdot \cos \theta_j) \right\}^2 \]

\[ = \frac{1}{2} \rho g \sum_{i} \sum_{j} \sum_{k} \sum_{l} \eta_{ij} \eta_{kl} S_i S_k \sin \theta_j \sin \theta_l \]

\[ \times \cos \left( a_{ij} + \epsilon_{ij} - \epsilon_{kl} - a_{ijkl} \right) \]

\[ a_{ijkl} = K_i \cos \theta_j - K_k \cos \theta_l \]

\[ S = \sqrt{1 + 2K_i / \sinh(2K_i)} \]
Quantitative Risk Analysis of VLFS Multiple Mooring Dolphins

Slowly Varying Wave Drift Force

Simple method

\[ F_y = \int_0^L f_y \, dx \]

\[ = \frac{1}{2} \rho g L \sum_i \sum_j \sum_k \sum_l \eta_{ij} \eta_{kl} S_i S_k \sin \theta_j \sin \theta_l \]

\[ \times P \cdot \cos\left(\omega_i - \omega_k \right) t + \epsilon_{ij} - \epsilon_{kl} - \alpha_{ijkl}\right) \]

\[ \alpha_{ijkl} \equiv \tan^{-1} \frac{1 - \cos (\alpha_{ijkl} L)}{\sin (\alpha_{ijkl} L)} \]

\[ P \left( \equiv \sqrt{\frac{2 - 2 \cos (\alpha_{ijkl} L)}{a_{ijkl} \frac{L}{L}}} \right) \]
Quantitative Risk Analysis of VLFS Multiple Mooring Dolphins

 failure probability

\[ P = \exp\left[-3.74763 \times (\theta - 3.49707)^{0.413556}\right] \]

\[ P = \exp\left[-2.9 \times (\theta - 3.49707)^{0.413556}\right] \]

\[ \theta \geq 90° \]

\[ \theta \leq 120° \]

Number of mooring dolphin on the long side